COMPARISON OF BACKSTEPPING AND SLIDING MODE CONTROL TECHINIQUES FOR A HIGH PERFORMANCE ACTIVE VEHICLE SUSPENSION SYSTEM

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Abstract: The objective of this paper is to present the design and implementation of an active vehicle suspension system that aims at reducing vibrations experienced by the driver. Cost effectiveness, ride comfort and robustness are major design parameters in developing the solution. A half car suspension system test rig is constructed for testing the three implemented controllers. Back stepping is used as the primary controller due to its ability to handle non linear systems. The adaptive sliding mode controller is implemented to improve robustness and to deal with non parametric actuation related uncertainties of the controller. A comprehensive comparison of the performance of a back stepping controller was experimented and tested against the proportional integral derivative (PID) and the adaptive sliding back stepping (ASB) controllers in a progressive incremental manner. The experimental results showed that the back stepping, ASB and the PID controllers reduced the sprung mass displacement up to 76.8 %, 71.3 % and 60.9 % respectively when compared to the passive system. The adaptive sliding mode controller performance shows adaptive properties as its performance improves with time. Although ride comfort has been improved, the quality of the suspension travel has been compromised. Matlab, Simulink and DSpace are used for the programming environment.

Keywords: adaptive controller, back stepping controller, ride comfort, robustness, sliding mode, uncertainties.

1. INTRODUCTION

This paper serves as a discussion of the development of an Active Vehicle Suspension System (AVSS). The demand for these systems is increasing due to damping vibrations affecting the driver's health and the lifetime of the vehicle Ahmed and Purdy (2012). Vibrations resulting in driver discomfort are due to uneven road surfaces such as humps and potholes. A high performance AVSS requires the development of an advanced controller that achieves robustness and can handle parametric and non parametric uncertainties Zhou and Doyle (1998).

The project requires the construction of a mechanical test rig and the implementation of an advanced controller that demonstrates reduced perturbation impact. The solution should be cost effective and achieve improved driver comfort for different road disturbances. The designed controller should be capable of handling a highly non-linear suspension system, varying system conditions and uncertainties with unknown bounds. The major project constraints are time and budget limitations. The duration of the project is six weeks, therefore limiting the complexity of the solution. Due to budget constraints the choice of components should attempt to minimise the cost of constructing the mechanical test rig. Matlab, Simulink and Dspace are utilised as programming environments to implement the controllers and to transmit signals to the physical system.

The structure of this paper is as follows. Section 2 discusses suspension system test rigs and controller techniques. System modelling of the constructed test rig is computed in Section 3. The back stepping and adaptive sliding back stepping controller techniques to be applied on the mechanical test-rig are designed in Section 4. The obtained experimental results from tests conducted on the test rig are shown in Section 5. Critical analysis on the active suspension system's strengths, limitations, trade-offs and future improvements is discussed in Section 6. The paper's conclusion is in Section 7.

2. BACKGROUND

Vehicles such as mining dump trucks require high performance active suspension systems as they consists of heavy tyres (unsprung mass) and high load variations (sprung mass).An AVSS consists of a mechanical suspension system and a controller. To analyse the system's performance, a controller is applied to a developed mechanical test-bed. The controller receives mass displacement signals from the physical system and determines the required actuation force to ensure improved ride comfort and road handling capabilities.

2.1 Suspension system test-rig

In automotive control engineering, the demand of advanced controllers is increasing so as to achieve the desired driver trajectory. An active suspension system consists of force generating components such as actuators, springs and dampers. These forces act on the vehicle's body and wheels to produce displacement variations. Suspension system test rigs and controllers are frequently used for analysing ride comfort, road handling and suspension deflection concepts Huang and Lin (2004). The most frequently used active suspension system test beds are quarter cars, as they are simpler to construct and model when compared to half car and full car test beds. Although half car test beds are more costly and time consuming to construct they are used to achieve a more practical and accurate model for testing implemented controllers. Compared to quarter cars, half car models have higher degrees of freedom and also represent the pitch motion dynamics of the vehicle E. Venkateswarulu and Seshadri (2014). In A. Kruczek and Honcu (2010) a quarter car test bed is constructed consisting of a hydraulic power source as an input road disturbance and a linear electric motor for actuation. The electric motors convert electrical energy to linear mechanical motion that ensures that the car's body experiences reduced displacement A. Kruczek and Honcu (2010). Controllers are applied to suspension system test rigs to instruct the actuators on how much force they need to produce to achieve improved ride comfort.

2.2 Controller Techniques

Control methods are based on several control techniques such as optimal, classical, quantitative feedback and robust control. Suspension system components such as springs and dampers have non-linear components and varying parameter values, therefore the considered controllers should be able to handle system non-linearities, robustness and uncertainties. Back stepping, sliding mode, adaptive back stepping, Proportional Integral Derivative (PID) and state feedback using pole placement types of controllers are discussed in this section.

Back-stepping controller: The back stepping controller design technique has the ability to be exploited to satisfy the conflicting concepts of ride quality and suspension travel Huang and Lin (2004). This control methodology involves the use of linear filters whose input is the displacement of the wheel and the effective bandwidth. The regulated variable is defined as the relative displacement between the vehicle's body and the wheel, for the closed loop system such that the zero dynamics of the system can be avoided. The back stepping controller ensures that the closed loop system is capable of focusing on different control objectives for different operating range Lin and Kanellakopoulos (1997). Using the Lyapunov theory, stability is achieved since energy concepts state that the motion of a stable mechanical system decreases all the time Slotine (1991). In Bassari and Hamzah (2007), the back stepping control method is used on a non linear AVSS to optimise between the force transmitted to the driver and the tire road contact for improved car handling.

Sliding mode controller: Sliding mode controllers are used to achieve robustness in non linear mechanical systems. This control approach operates on the principle of applying a discontinuous control signal that results in the system sliding along the cross section of the system's regular response. This results in alterations on the dynamics of the system. This controller provides a systematic approach to maintaining stability and consistent performance Slotine (1991). The limitation of this control approach is that it requires knowledge of the bounds of the system uncertainties and it can also result in chattering. In Deshoande and Bhaskara (2012), sliding mode control method is used on an AVSS using a disturbance observer. Although this controller did not use the states of the sprung mass and did not require the bounds of the uncertainty, it improved the ride comfort for different road disturbances. Limitations of this solution are that although it achieved ride comfort, it has no control the road handling capability and the suspension deflection Deshoande and Bhaskara (2012).

Adaptive back-stepping: Adaptive controllers are useful in handling system parametric and non parametric uncertainties. Unlike sliding mode controllers, this control method eliminates the need to know system parameter bounds. The structure of an adaptive controller is similar to a robust controller but in addition, the model is updated during operation, based on the measured performance Slotine (1991). Conglomerate use of adaptive and sliding mode controllers can achieve robustness for varying system conditions. In Chingozha and Nyandoro (2014), Adaptive Sliding Back-stepping (ASB) is used on a quad rotor to achieve a controller that does not require prior knowledge of the upper bounds of the uncertainty. This controller can be used for systems with a strict feedback form with matched uncertainties. Simulations in Chingozha and Nyandoro (2014) show that this control method can produce global asymptotic tracking of the desired trajectory.

PID controllers: The PID controllers are designed to improve the settling time, rise time overshoots and eliminate the steady state error in control systems Valibhav and Shrikant (2013). This controller is based on a feedback mechanism and achieves the desired response by tuning the K_p,K_i and K_d gains. Tuning the gain can improve the system's time domain characteristics. In Ekoru and Pedro (2011), a two-loop PID controller is implemented for suspension travel in a half-car active vehicle suspension system with four degrees-of-freedom and non-linearities. The two-loop system consists of an inner PID hydraulic actuator force and an outer PID suspension travel control loop. The simulation results showed improvements for the active suspension system when compared to the passive system with the same model parameters Ekoru and Pedro (2011).

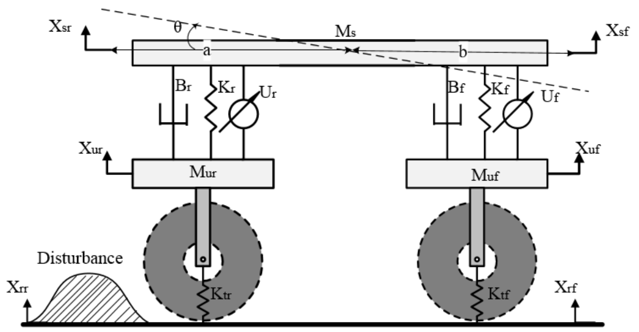

Fig. 1. Model of a half car test rig.

2.3 *Project solution*

A half car mechanical suspension system is designed and constructed as a test rig for the implemented back-stepping, PID and ASB controllers. The back-stepping controller design is chosen as the primary solution for this project due to its ability to be exploited to satisfy the conflicting concepts of ride quality and suspension travel.

3. SYSTEM MODELLING

The physical system has force generating components such as springs, dampers and actuators as shown in Fig. 1. Springs and dampers consists of linear and non-linear components. The system modelling only consists of the linear components to limit the complexity of the formulation. The modelling also assumes that the vehicle driver is seating on the center of the half car model.

3.1 *Physical system*

Table 1. Parameters for the physical system

Parameter	Value	Units
Sprung mass (M_s)	1.2	kg
Front unsprung mass (M_uf)	0.45	kg
Rear unsprung mass (M_ur)	0.45	kg
Front suspension spring (K_f)	901	N/m
Front suspension damper (B_f)	12	Ns/m
Front tire stiffness (K_tf)	1527	N/m
Rear suspension spring (K_r)	901	N/m
Rear suspension damper (B_r)	12	Ns/m
Rear tire stiffness (K_tr)	1527	N/m

3.2 *Mathematical model*

The motion equation results from applying Newton's second law to the vehicle body and wheels. The state variables are assigned as shown in eq. 1. The resulting equations for the front sprung mass, rear sprung mass, front unsprung mass and forces acting on the front sprung mass are shown in eq. 2 to eq. 5 respectively. eq. 6 is then derived from eq. 1.

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (x_{sf}, \dot{x_{sf}}, x_{uf}, \dot{x_{uf}}, x_{sr}, \dot{x_{sr}}, x_{ur}, \dot{x_{ur}}) \quad (1)$$

$$\ddot{x}_{sf} = \ddot{x}_1 = 1/m_s[-K_f(x_{sf} - x_{uf}) - B_f(\dot{x}_{sf} - \dot{x}_{uf}) - K_r(x_{sr} - x_{ur}) - B_r(\dot{x}_{sr} - \dot{x}_{ur}) + f_f + f_r] \quad (2)$$

$$\ddot{x_{sr}} = \ddot{x_3} = 1/m_s[-K_f(x_{sf} - x_{uf}) - B_f(\dot{x_{sf}} - \dot{x_{uf}}) - K_r(x_{sr} - x_{ur}) - B_r(\dot{x_{sr}} - \dot{x_{ur}}) + f_f + f_r] \quad (3)$$

$$\ddot{x_{uf}} = \ddot{x_5} = 1/m_u[-K_f(x_{sf} - x_{uf}) - B_f(\dot{x_{sf}} - \dot{x_{uf}}) - K_{tf}(x_{uf} - x_{rf}) - f_f] \quad (4)$$

$$\ddot{x_{ur}} = \ddot{x_7} = 1/m_u[-K_f(x_{sr} - x_{ur}) - B_f(\dot{x_{sr}} - \dot{x_{ur}}) - K_{tr}(x_{ur} - x_{rr}) - f_r] \quad (5)$$

$$\dot{x_1} = x_2,\ \dot{x_3} = x_4,\ \dot{x_5} = x_6,\ \dot{x_7} = x_8 \quad (6)$$

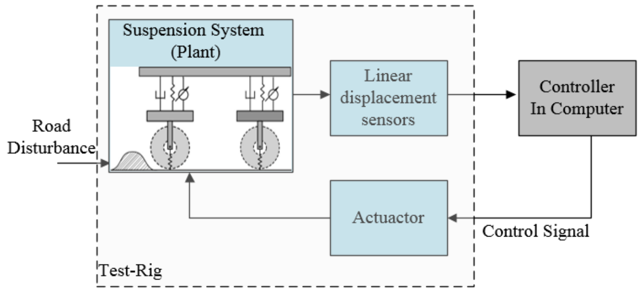

Fig. 2. Components of the active suspension system.

Due to construction of the physical system, the pitch dynamics will be assumed to be negligible since there is no firm mechanical coupling between the front and rear body(sprung mass).

3.3 *Active suspension system components*

FIG. 2 shows the main components of an AVSS. The physical test rig consists of the plant, four displacement sensors and two actuation motors. The displacement sensors measure the change in displacement for the front and rear sprung and unsprung mass. The signal from these sensors is sent to the controller using Dspace. The controller then computes the required actuation force required to achieve the desired driver trajectory. Once this force has been computed by the front and rear controllers, a signal is sent to the servo motors. The servo motors produce rotational motion, which converts to linear displacement through the use of a rack and pinion. Therefore when the vehicle encounters a road disturbance, the signals from the sensors will enable the computer system to compute the change in displacement in that particular actuator and result in a response that compensates for the change in displacement. The constructed mechanical test rig is shown in Fig. 3.

4. CONTROLLER TECHNIQUES

Back-stepping is implemented as the primary controller for the half car suspension system. Sliding mode and adaptive back-stepping are then conglomerated to improve robustness and to deal with parametric uncertainties associated with the controller input.

4.1 *Back stepping controller*

Back stepping is designed and implemented for the front and rear part of the suspension system. This controller

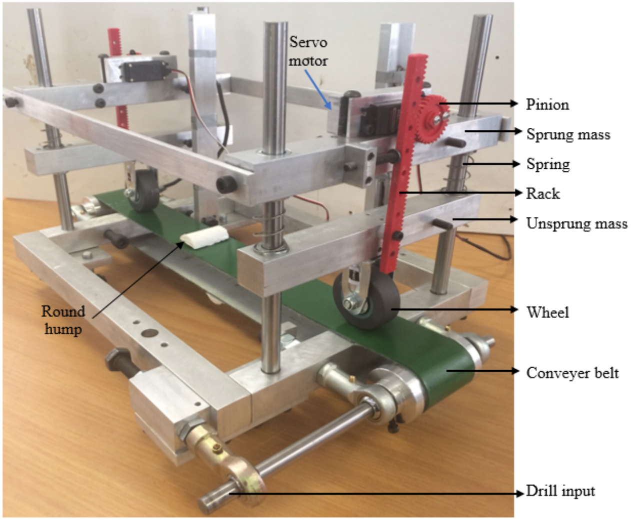

Fig. 3. Constructed mechanical test rig.

stabilises the non-linear dynamic suspension system by using the Lyapunov stability theory.

Front controller design: The regulating variable for the front part is selected as shown in Eq. 7 Huang and Lin (2004). The derivative of the regulating variable is shown in Eq. 8. The linear design filter is as shown in Eq. 9.

$$z_1 = x_1 - \overline{x}_3 \tag{7}$$

$$\dot{z_1} = x_2 + \epsilon_1(x_1 - x_3) - \epsilon z_1 \tag{8}$$

$$\overline{x}_3 = \frac{\epsilon_1}{(s + \epsilon_1)} x_3 \tag{9}$$

For small values of ϵ_1, $\overline{x}_{uf}$ is a low pass filter and $z_1 = x_{sf}$. The oscillations are eliminated and the driver ride comfort is improved whereas the suspension deflection quality is compromised. If ϵ_1 is large high frequencies pass through and $z_1 = x_{sf} - x_{uf}$. The first virtual control variable is x_2 and the error variable is as shown by Eq. 10. $\phi(x)$ is the desired x_2 value. The derivative of the error variable is shown in Eq. 11.

$$z_2 = x_2 - \phi_1(x) \tag{10}$$

$$\dot{z_2} = \dot{x_2} - \dot{\phi_1}(x) \tag{11}$$

$$\phi_1(x) = -\epsilon(x_1 - x_3) - c_1 z_1 \tag{12}$$

The candidate Lyapunov function for stabilising z_1 is shown in Eq. 13. It is a locally positive definite function.

$$V_f(z) = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \tag{13}$$

$$\dot{V_f}(z) = -(c_1 + \epsilon_1) z_1^2 - c_2 z_2^2 \tag{14}$$

Where c_1 is the design constant and $c_1 > 0$, that ensures the derivative of the Lyapunov function is negative definite

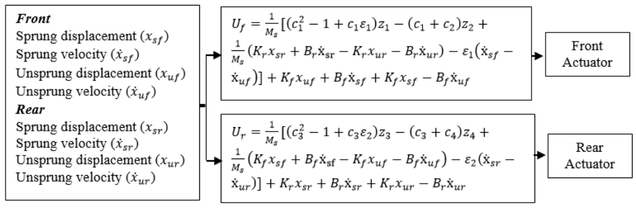

Fig. 4. Front and rear back-stepping controller equations.

as shown in Eq. 14. c_2 is the second design constant and $c_2 > 0$. The remaining states are the zero dynamics, these are found to be exponentially stable for $\epsilon_1 > 0$ Lin and Kanelakopoulos (1995). The resulting controller equation is shown in Fig. 4.

Rear controller design: The regulating variable for the rear part of the suspension system is shown in Eq. 15:

$$z_3 = x_{sr} - \overline{x}_{ur} \tag{15}$$

Where the linear design filter is shown in Eq. 16 is:

$$\overline{x}_{ur} = \frac{\epsilon_2}{s + \epsilon_2} x_{ur} \tag{16}$$

The rear controller error state variable is shown in Eq. 17.

$$z_4 = x_6 - \phi_2(x) \tag{17}$$

$$\phi_2(x) = -\epsilon_2(x_5 - x_7) - c_3 z_3 \tag{18}$$

The candidate Lyapunov function for stabilising z_3 is in Eq. 19. It is a locally positive definite function. The time derivatives are negative definite as shown in Eq. 20. The design constant are $c_1 = c_3 = 6, c_2 = c_4 = 1000$ and $= \epsilon_1 = \epsilon_2 = 0.0001$.

$$V_r(z) = \frac{1}{2} z_3^2 + \frac{1}{2} z_4^2 \tag{19}$$

$$\dot{V_r}(z) = -(c_3 + \epsilon_2) z_3^2 - c_4 z_4^2 \tag{20}$$

The rear and front controller inputs are shown in Fig. 4.

4.2 Sliding mode controller design

The sliding mode controller is implemented to improve robustness within given bounds. The sliding mode technique is applied to ensure that z_2 is zero Chingozha and Nyandoro (2014). The control task is to obtain an input (u) that meets the criterion in Eq. 21.

$$\frac{1}{2} . \frac{dz_2^2}{dt} \leq -\eta \left| z_2 \right| \tag{21}$$

The sliding mode techniques divides the input to the equivalent control (u_{equiv}) which ensures that for the nominal z_2 dynamics, $\dot{z_2}$ is always zero and the switching input (u_{switch}) which ensures Eq. 21 is met. From Eq. 11, the general expression of $\dot{z_2}$ is derived and shown in Eq. 22.

$$\dot{z_2} = \dot{\phi}_1(x) + f(x) + gu \tag{22}$$

Therefore u_{equiv} is as shown in Eq. 23, if f(x) and g are unknown. The switching control is assumed to be as shown in Eq. 24.

$$u_{equiv} = \frac{1}{g}[\dot{\phi}_1(x)] \tag{23}$$

$$u_{switch} = -\Upsilon(z_1, z_2)sign(z_2) \tag{24}$$

The full sliding back stepping mode control law for the front controller is in Eq. 25:

$$U_f = \frac{1}{g_f}[\dot{\phi}_1(x)] - \Upsilon(x_1, z_1, z_2)sign(z_2) \tag{25}$$

Following the same procedure, the rear controller is in Eq. 26:

$$U_r = \frac{1}{g_r}[(\dot{\phi}_2(x)] - \Upsilon(x_2, z_3, z_4)sign(z_4) \tag{26}$$

4.3 Adaptive back stepping controller design

Adaptive back stepping is implemented as it does not require any knowledge about the bounds of the uncertainty. Assuming there exists a positive constant d such that $\Upsilon(x_1, z_1, z_2) < d$ for all time, therefore estimation error is $\tilde{d} = d - \hat{d}$. The candidate Lyapunov function is shown in Eq. 28. The selected adaptive law is shown in E1. 30.

$$\upsilon(x_1, z_1, z_2) = \tilde{d} \tag{27}$$

$$V = \frac{1}{2}z_2^2 + \frac{\tilde{d}^2}{2\psi} \tag{28}$$

$$U_f = \frac{1}{g}[\dot{\phi}_2(x)] - \tilde{d}sign(z_2) \tag{29}$$

$$\hat{d} = \psi|z_2| \tag{30}$$

The control input obtained from the ASB is implemented and compared to other controllers. ASBs limitation is that it generates a relatively high control input as shown in Fig. 8.

4.4 PID controller

The PID gains are tuned to maximise on ride comfort in the Simulink implementation. The selected K_p, K_i and K_d are 700, 30 and 10 respectively. These result in the best performance for tested road disturbances.

5. TESTING RESULTS

Results were obtained for two types of road disturbance which are the square and round hump. The designed back stepping, PID and ASB controllers are tested for the front and rear parts of the system. The sprung mass displacement results for the implemented controllers on both of the road disturbance are shown in Fig. 5 to Fig. 8. The analysis compares the magnitude of the displacement of the sprung mass, and not their response time as the controllers were not tested at the same time.

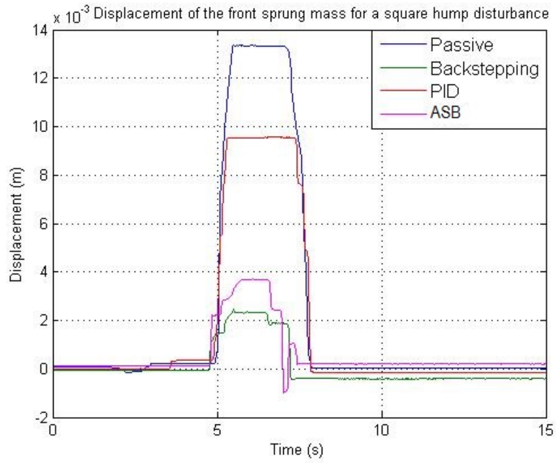

Fig. 5. Front sprung displacement for the square hump.

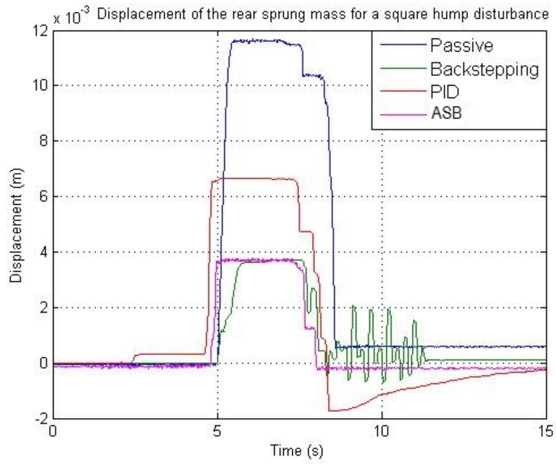

Fig. 6. Rear sprung displacement of the square hump.

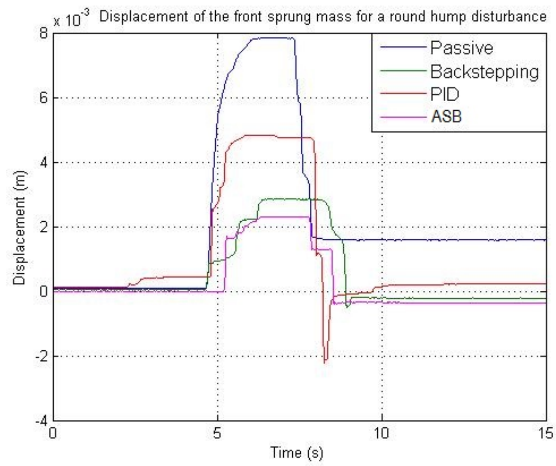

Fig. 7. Front sprung displacement of the round hump.

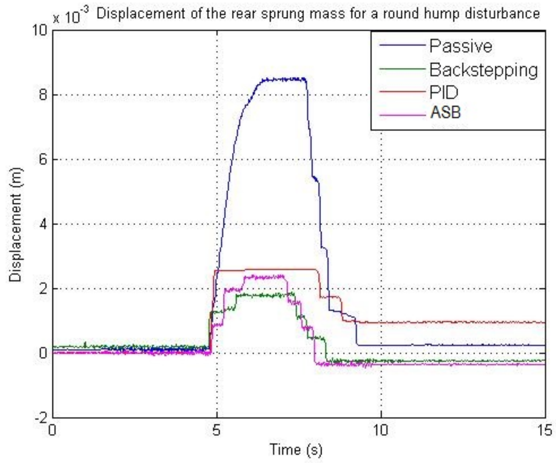

Fig. 8. Rear sprung mass displacement of the round hump.

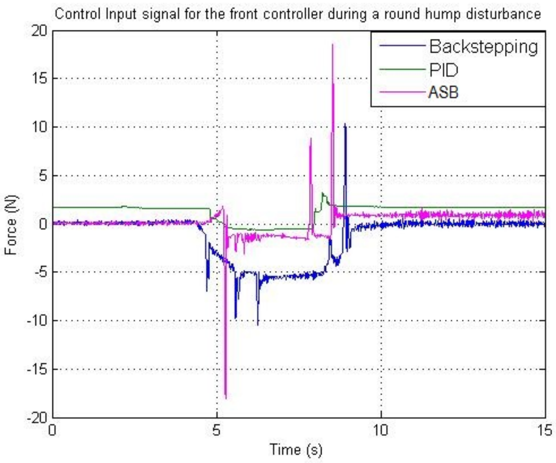

Fig. 9. Control input signals.

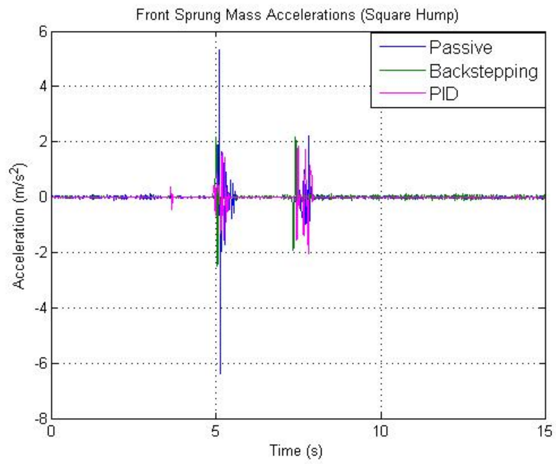

Fig. 10. Front sprung acceleration for the square hump.

6. CRITICAL ANALYSIS

A robust test rig was constructed within the time constraints. The construction of the physical model is cost effective as shown in Appendix A, Figure 2. The use of rotational servo motors with rack and pinions to produce linear motion was more cost effective than using linear mo-

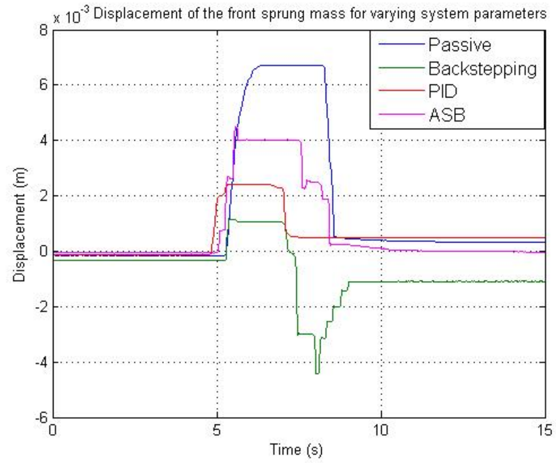

Fig. 11. Robustness demonstration for varying parameters.

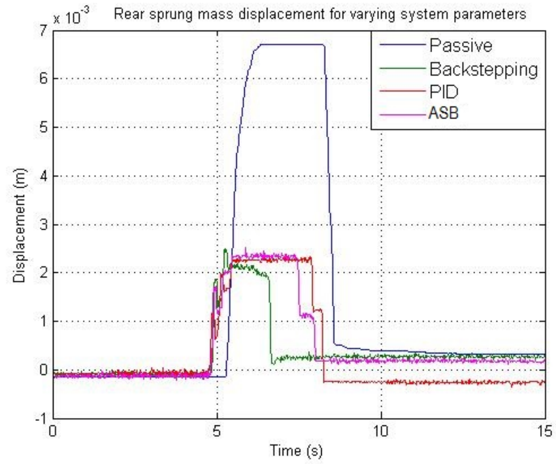

Fig. 12. Robustness demonstration for varying parameters.

tors such as the Linear Variable Differential Transformer (LVDT) as these where were costly. Although sliding potentiometer displacement sensors where selected for being cost effective, they introduced a lot of friction into system thereby leading to an over damped system. The size of the developed test bed is portable for testing purposes. Due to the loose connection of the front and rear parts of the system, the pitch dynamics where neglected. The testing results show that the designed controllers achieve improved ride comfort when compared to the passive system. Therefore road disturbances will not have adverse impact on the vehicle driver. Table 2 shows a summary of the results obtained in Fig. 4 to Fig. 7. In overall, the back stepping controller shows the best performance as compared to the PID and ASB controllers.

Table 2. Controller performance when compared to the passive system.

	Square Hump		Round Hump	
	Rear (%)	Front (%)	Rear (%)	Front (%)
PID	48	35	60.9	37.5
Back stepping	68.3	73.2	76.8	62.5
ASB	68.3	63	68.2	71.3

Fig. 9 shows that the sprung mass does not experience much accelerations, this is due to the physical system being over damped. However, to reduce the signal noise experienced and depicted in Fig. 9, filtering techniques should be implemented. The sliding adaptive back stepping controller performance is demonstrated in Fig. 10 when there are changes introduced to the system's spring constants. The ASB model is updated during operation, based on the measured performance. The front sprung mass accelerations overshoot can be minimised by implementing control methods with constraints.

The use of a half car model instead of a quarter car compromises cost effectiveness but achieves a more accurate model of the mechanical vehicle. Rattle space usage and driver ride comfort are the major design trade-offs. The project can be improved by using a full car test-bed as this would be a more practical test rig. Cameras and sensors can be used for preview so that the controller can counteract the road unevenness at the right time.

7. CONCLUSION

A half car test bed is constructed and a back stepping, PID and ASB controllers are applied to isolate the driver from the uneven road disturbances. The experimental results show that the back stepping, ASB and the PID controllers reduce up to 76.8 %, 71.3 % and 60.9 % respectively when compared to the passive system. The software tools used are Matlab, Simulink and Dspace.

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