

# An evolutionary strategy for multiobjective reinsurance optimization - A case study



**Juan G. Villegas**

Profesor Asociado

Departamento de Ingeniería Industrial  
Universidad de Antioquia - Medellín (Colombia)

**Sebastian Roman**

Instituto Tecnológico Metropolitano de Medellín

**Andrés M. Villegas**

CEPAR

Centre of Excellence in Population Ageing Research  
University of New South Wales– Sydney (Australia)



# Agenda

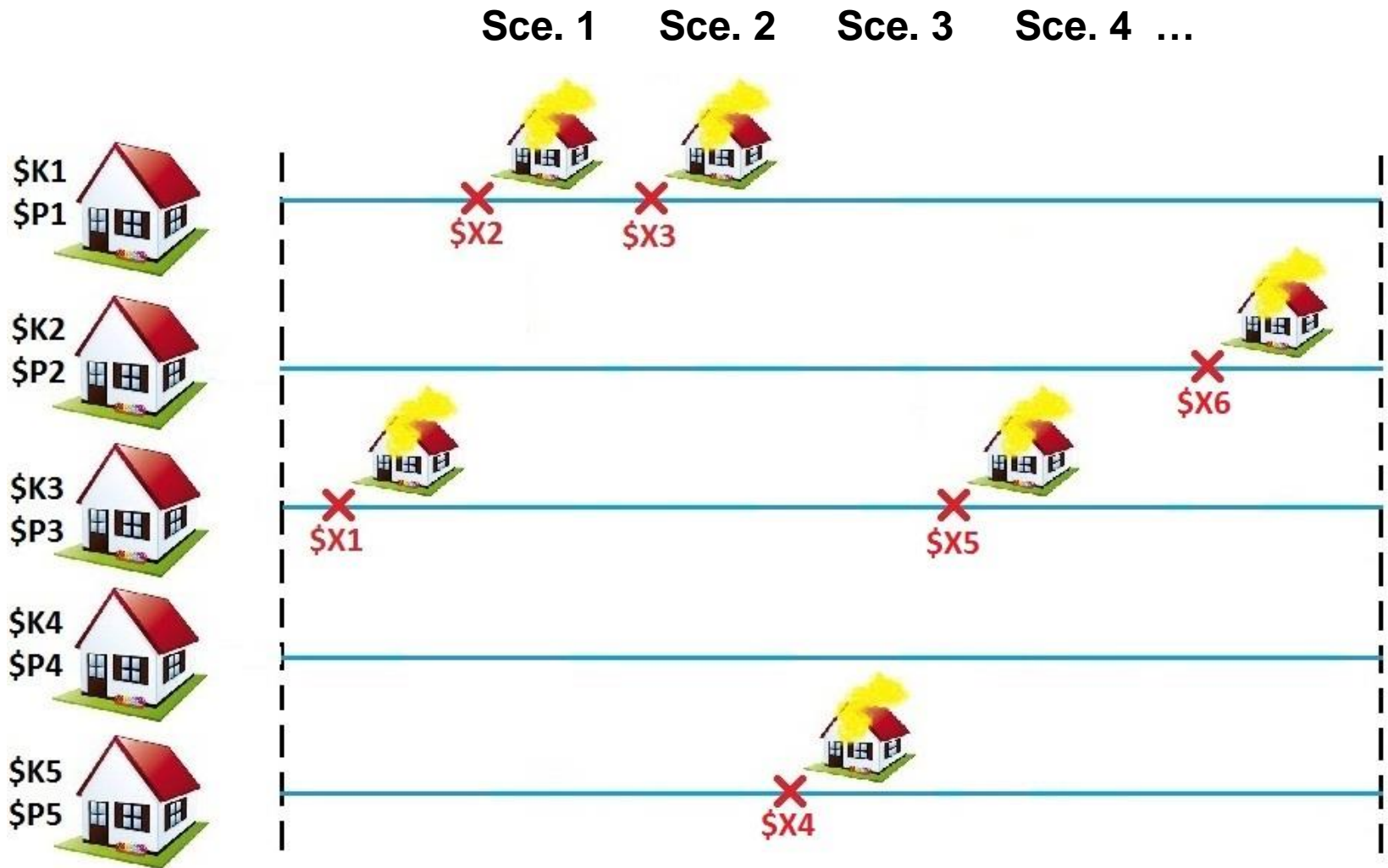
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1. Introduction and problem definition
2. Solution method
3. Case study

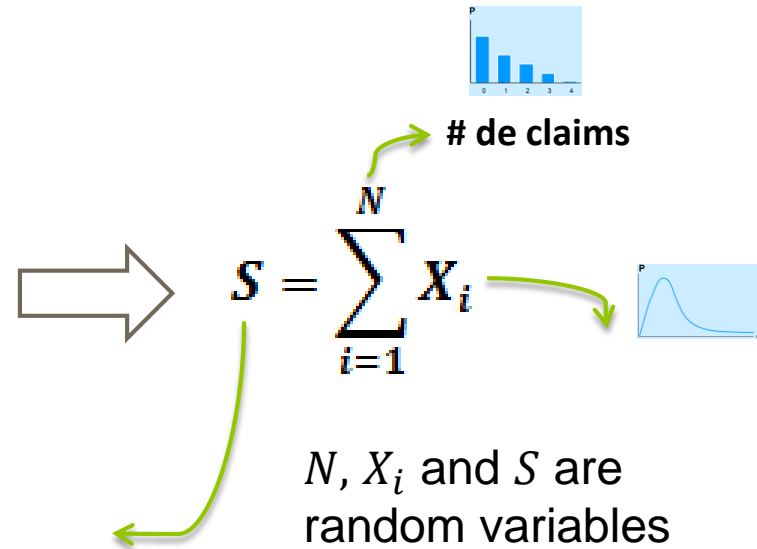
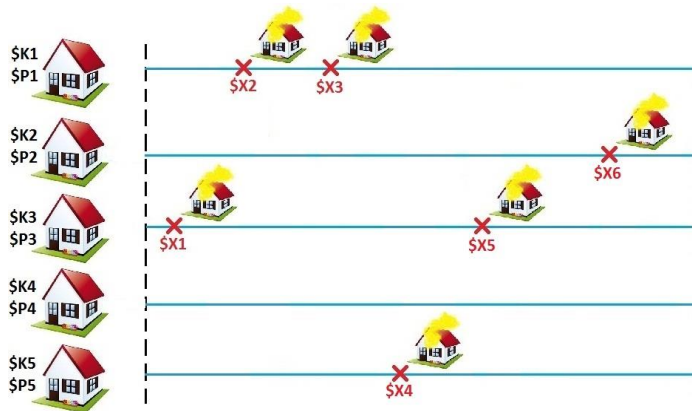


# Introduction and problem definition

# Insurance business



# Risk of an insurance portfolio



$S$ : Cumulated losses of the insurance company

# Reinsurance contracts (R/A)

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The insurance company transfers portions of its risk to other companies (the reinsurer)

$$S = S_R + S_C$$

$S_R$ : retained risk

$S_C$ : ceded risk

**Risk transfer functions**

$$S_C = F_1 \left( \sum_{i=1}^N F_2(X_i) \right) \longrightarrow S_R = S - S_C$$

$F_1$ : global transfer function

$F_2$ : local transfer function

**Reinsurance premium (cost of the reinsurance):**

$$P_{R/A} = \Pi(S_C)$$

$\Pi$ : Pricing principle

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# Reinsurance optimization problem (ROP)

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Financial result of the insurance company

$$U = P - S + S_C - \Pi(S_C)$$

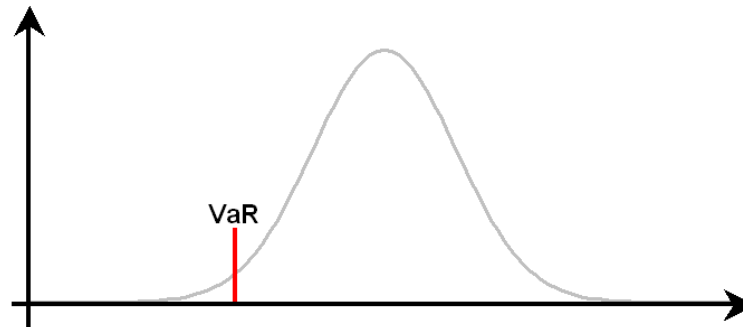
$P$ : insurance premiums,

$S$ : total losses,

$S_C$ : ceded losses

$\Pi(S_C)$ : reinsurance premium

**ROP: Find  $F_1$  and  $F_2$  that maximize  $E(U)$  and minimize a measure of risk of  $U$ , for example  $VaR_{0.99}(U)$**



# Literature overview

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Reference	Method	Comments
Oesterreicher et al (2006)	NSGA-II and $\epsilon - MOEA$	First MOEAs for R/A optimization
Mitschele et al (2007)	NSGA-II and $\epsilon - MOEA$	Practical application fire risks
Carmona et al (2013)	Population based incremental learning (PBIL)	Layered reinsurance - Scalarization approach
Carmona et al (2014)	PSO and differential evolution (DE)	Layered reinsurance - Scalarization approach
Salcedo Sanz et al (2014)	Evolutionary strategy and PSO	Mono-objective optimization, simple reinsurance contracts
Brown et al (2014)	Parallel multiobjective PBIL	Computing time reduction, no scalarization
Carmona et al (2015)	Parallel multiobjective DE	Computing time reduction, no scalarization





# Literature overview

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Reference	Method	Comments
Oesterreicher et al (2006)	NSGA-II and $\epsilon - MOEA$	First MOEAs for R/A optimization, high mutation probabilities
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Carmona et al (2013)	Population based incremental learning (PBIL)	Layered reinsurance - Scalarization
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**We propose a multiobjective evolutionary strategy to solve the ROP**

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# Classical reinsurance contracts/ Proportional reinsurance

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- ▶ Based on the individual losses  $S$ :

$$S = \sum_{i=1}^N X_i$$

- ▶ **Proportional reinsurance:** A portion of the loss is transferred to the reinsurer

## Quota share reinsurance (CP)

$$X_{i_c} = (1 - a_i)X_i \longrightarrow S_c = \sum_{i=1}^N X_{i_c}$$

$a_i$ : Retention value (usually the same for all  $i$ )

*Example*

$$X_i = 100$$

$$a_i = 0.25$$

$$X_{i_c} = (1 - 0.25)100 = 75$$

# Classical reinsurance contracts/ Proportional reinsurance

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- ▶ Based on the individual losses  $S$ :

$$S = \sum_{i=1}^N X_i$$

- ▶ **Proportional reinsurance:** A portion of the loss is transferred to the reinsurer

## Surplus reinsurance (Exc)

$$X_{ic} = (1 - a_i)X_i \quad \text{with} \quad a_i = \min\left(\frac{L}{K_i}, 1\right)$$

**L**: **Line of the reinsurance**

$K_i$ : Insured value of loss  $i$

### *Example*

$$X_i = 100$$

$$L = 90$$

$$K_i = 180$$

$$a_i = \min\left(\frac{90}{180}, 1\right) = 0.5$$

$$X_{ic} = (1 - 0.5)100 = 50$$



# Classical reinsurance contracts/ Proportional reinsurance

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- ▶ Based on the individual losses  $S$ :

$$S = \sum_{i=1}^N X_i$$

- ▶ **Non proportional reinsurance:** Truncates the losses

## Excess of Loss reinsurance (XL)

$$X_{ic} = \max(X_i - P, 0) \longrightarrow S_c = \sum_{i=1}^N X_{ic}$$

**P: Priority of the excess of loss**

*Example*

$$X_i = 100$$

$$P = 90$$

$$X_{ic} = \max(100 - 90, 0) = 10$$

# Classical reinsurance contracts/ Proportional reinsurance

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- ▶ Based on the individual losses  $S$ :

$$S = \sum_{i=1}^N X_i$$

- ▶ **Non proportional reinsurance:** Truncates the losses

**Stop Loss reinsurance (SL):** applies to the aggregate losses

$$S_c = \max(S - R, 0)$$

**R: Priority of the stop loss**

*Example*

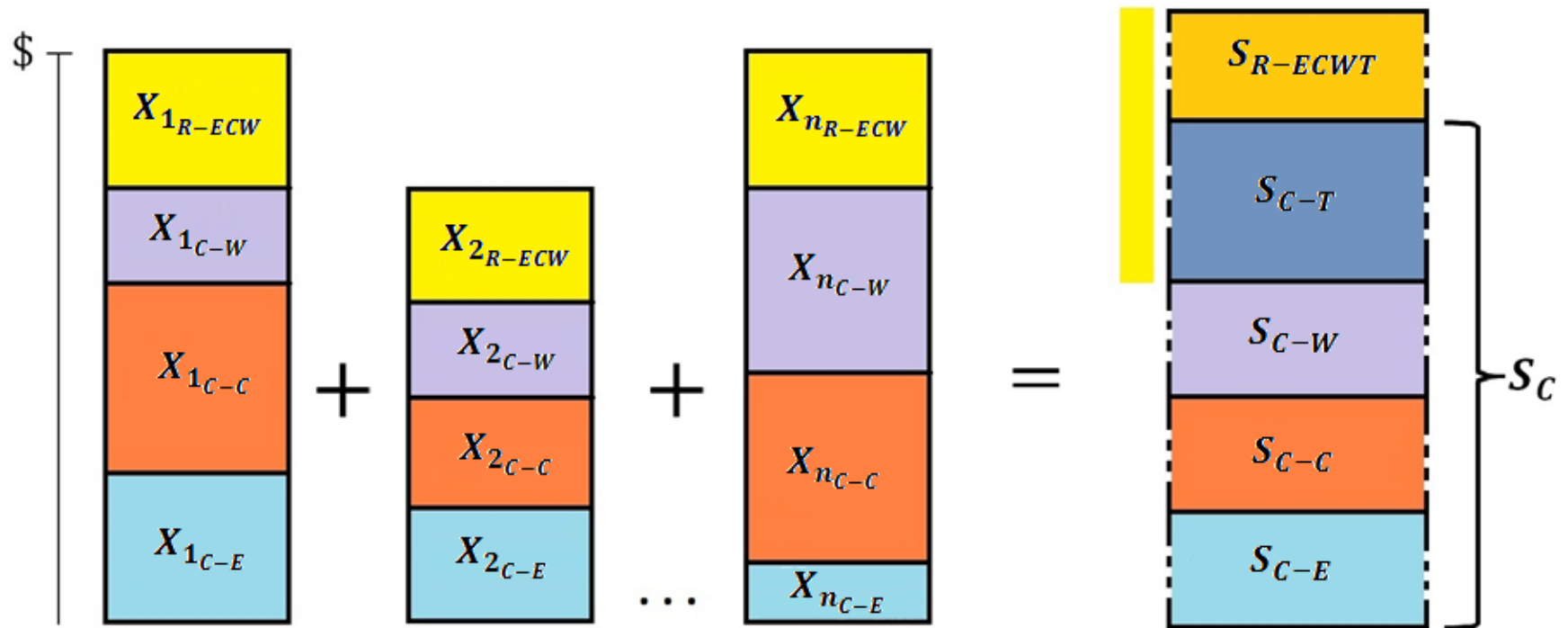
$$S = 1000$$

$$R = 800$$

$$S_c = \max(1000 - 800, 0) = 200$$

# Ceded and retained risk

$$S_C = S_{C-Exc} + S_{C-CP} + S_{C-XL} + S_{C-SL}$$



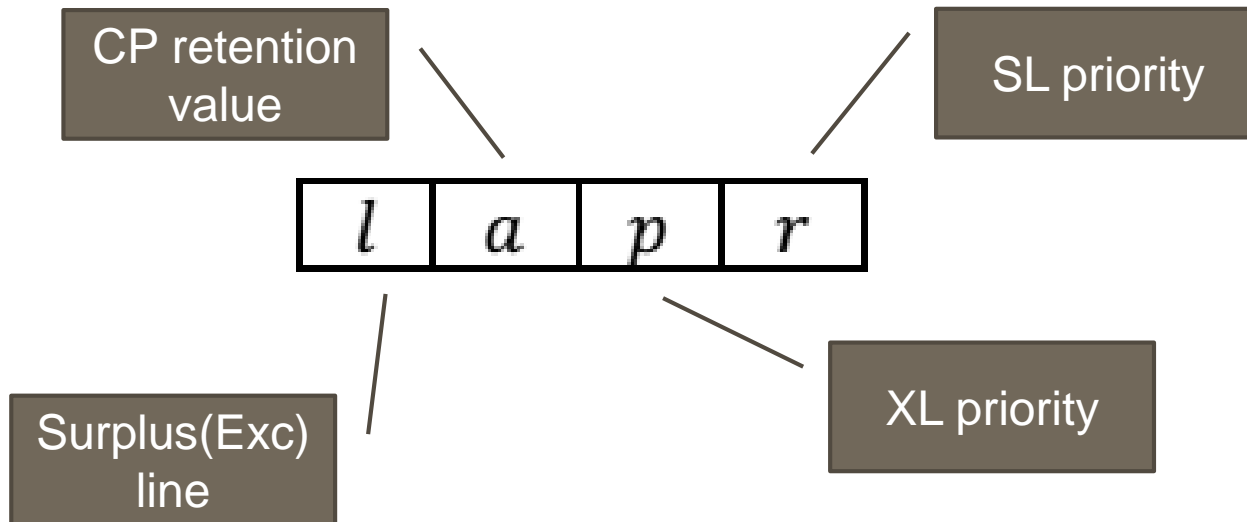


Solution method:  
Evolutionary strategy

# Solution representation

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- ▶ Solution representation





# Objective function

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$$U = P - S + S_c - \Pi(S_c)$$

- ▶ Minimize  $\Pi(S_c)$

$$f_1(l, a, p, r) = P_{R/A} = \Pi(S_c) = (1 + \lambda)E(S_c)$$

- ▶ Minimize the risk of the retained losses ( $S_R = S - S_c$ )

$$f_2(l, a, p, r) = VaR_\alpha(S_R) = F^{-1}(\alpha)$$

# Objective function calculation: since $N$ , $X_i$ and $S$ are random variables

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## ► Probabilistic modeling of the insurance portfolio

Individual loss for a claim,  $\mathbf{X}$ :  $F_X(x) = \int_{k=0}^{\infty} F_Z\left(\frac{x}{k}\right) c(k) \partial k$

Retained loss after reinsurance Exc,  $\mathbf{X}_{R-E}$ :  $F_{X_{R-E}}(x) = \int_{k=0}^L F_Z\left(\frac{x}{k}\right) c(k) \partial k + \bar{c}(L) F_Z\left(\frac{x}{L}\right)$

Retained loss after reinsurance Exc and CP,  $\mathbf{X}_{R-EC}$ :  $F_{X_{R-EC}}(x) = \int_{k=0}^L F_Z\left(\frac{x}{ak}\right) c(k) \partial k + \bar{c}(L) F_Z\left(\frac{x}{aL}\right)$

Retained loss after E, CP and XL,  $\mathbf{X}_{R-ECX}$ :

$$F_{X_{R-ECX}}(x) = \begin{cases} \int_{k=0}^L F_Z\left(\frac{x}{ak}\right) c(k) \partial k + \bar{c}(L) F_Z\left(\frac{x}{aL}\right); & \text{si } x < P \\ 1; & \text{si } x \geq P \end{cases}$$

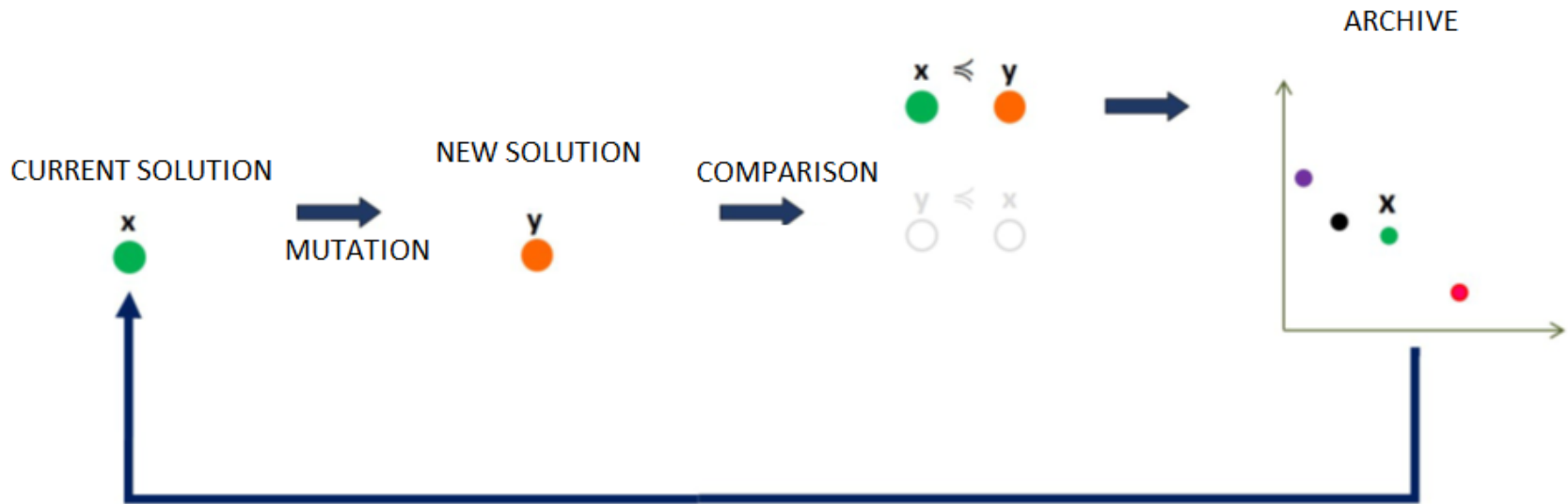
Aggregated losses after Exc, CP y XL,  $\mathbf{S}_{R-ECX}$ :

Aggregated retained losses after Exc, CP, XL y SL,  $\mathbf{S}_{R-ECXS}$   $F_{S_{R-ECXS}}(x) = \begin{cases} F_{S_{ECX}}(x); & \text{si } x < R \\ 1; & \text{si } x \geq R \end{cases}$

**We use numerical integration after fitting appropriate statistical distributions**

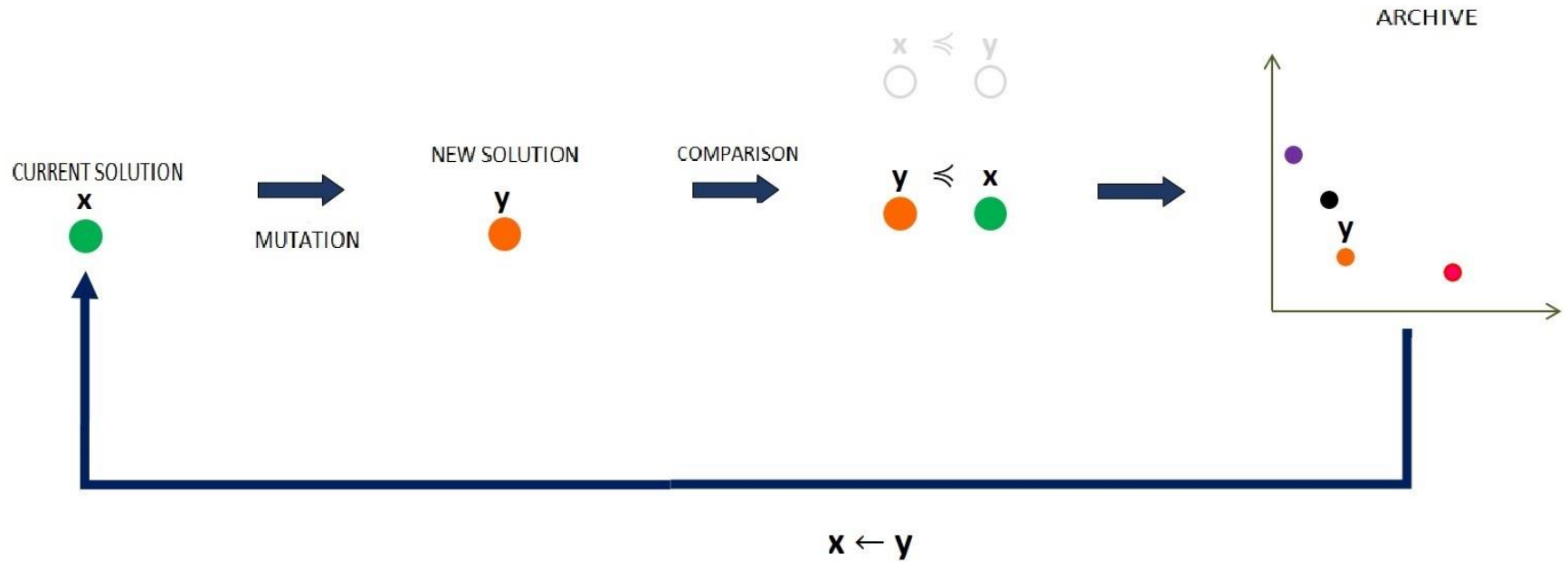
# A simple Pareto archived evolutionary strategy

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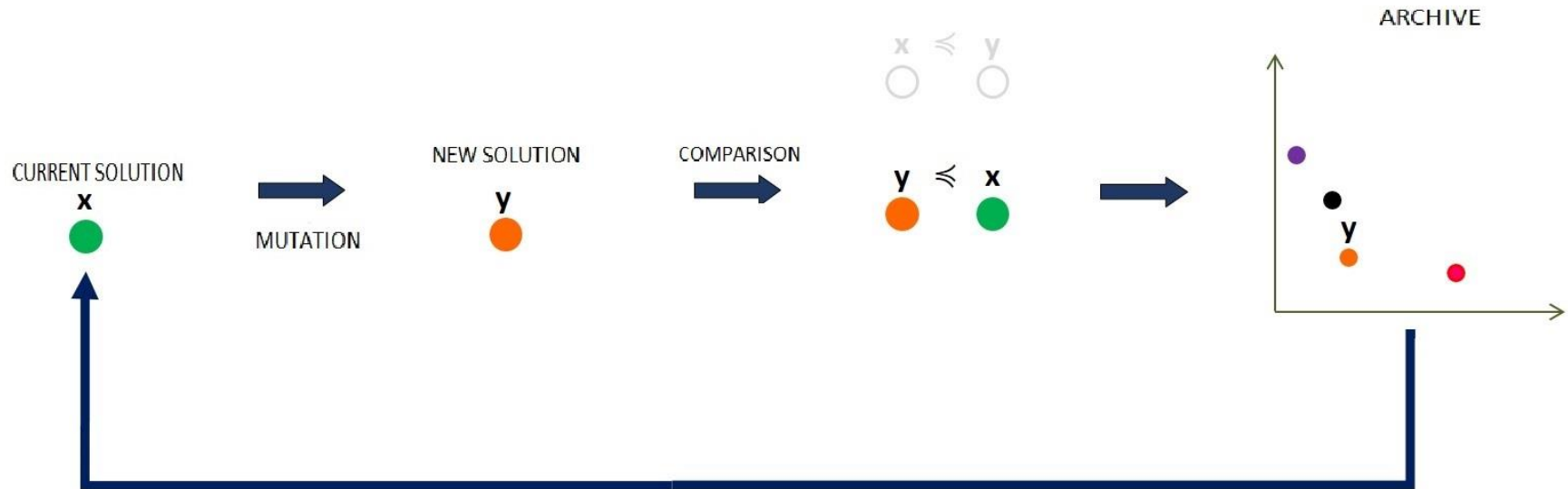
# A simple Pareto archived evolutionary strategy (ES)

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# A simple Pareto archived evolutionary strategy (ES)

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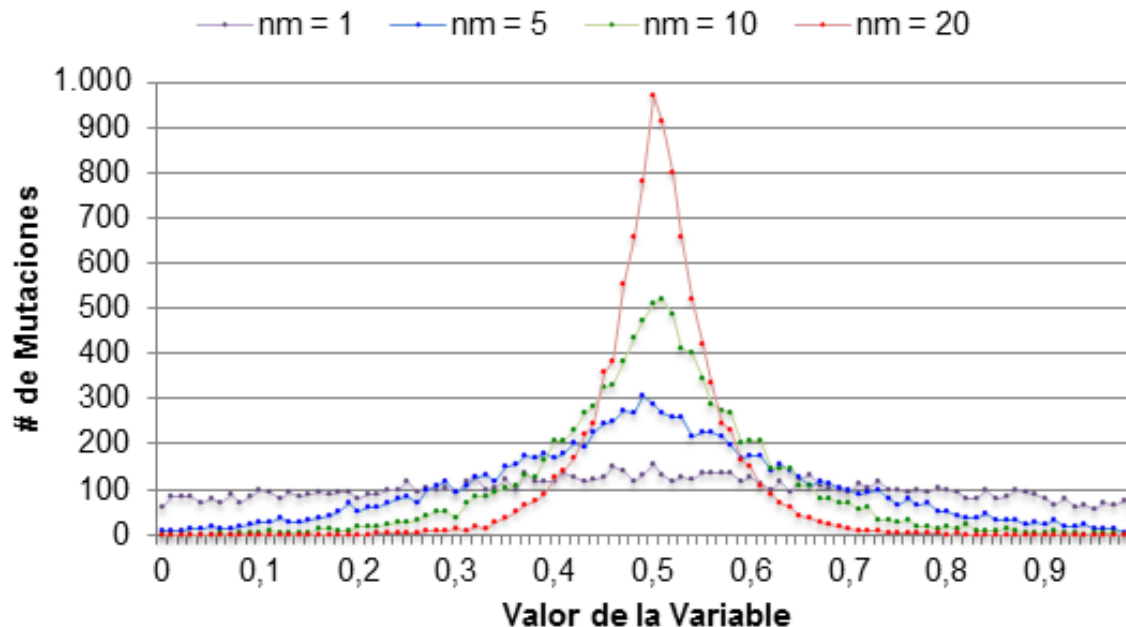
**RESTART FROM A NEW SOLUTION IN THE  
ARCHIVE AFTER 20 GENERATIONS WITHOUT  
IMPROVEMENT OF THE FRONTIER**

# Mutation operator

- ▶ Parameter based mutation operator (Deb & Goyal, 1996)

$$\bar{\delta} = \begin{cases} [2u + (1 - 2u)(1 - \delta)^{n_m+1}]^{\frac{1}{n_m+1}} - 1, & \text{si } u \leq 0.5 \\ 1 - [2(1 - u) + 2(u - 0.5)(1 - \delta)^{n_m+1}]^{\frac{1}{n_m+1}}, & \text{si } u > 0.5 \end{cases}$$

$$\delta = \min[(x - x_m), (x_M - x)] \quad y' = y + \bar{\delta}\Delta_{max} \quad \Delta_{max} = x_M - x_m$$



# Case study

# Implementation and case study

- ▶ Evolutionary strategy (ES) implemented in C#
- ▶ Decision support system for the Company
  - ▶ Results for mutation index  $n_m = 10$ , archive size: 50 and number of iterations of ES: 5000

**Programa Optimización R/A**

**Definición de la Cartera**  
Defina las variables aleatorias que definen el comportamiento siniestral de la cartera

Distribución Frecuencia de Siniestros: Poisson  
Parámetros Distr. de Frecuencia:  $\lambda = 100$

Seleccione el modo en que se definirá la distribución de la pérdida en un siniestro  
Mediante las distribuciones del Valor Expuesto (VE) y de Afectación al VE.

Distribución VE por Riesgo: LogNormal  
Parámetros Distr. VE por Riesgo:  $\mu = 5$ ,  $\sigma = 20$

Distribución Afect. al VE en un siniestro: Triangular  
Parámetros Distr. Afect. al VE en un siniestro: A=0, B=0,05, C=0,3

**Parámetros para la Optimización**  
Seleccione los tipos de reaseguro que desea considerar en la optimización  
 Excedente  Cuota Parte  Exceso de Pérdida por Siniestro (WXL)  Stop Loss (SL)

Seleccione el principio de costeo de reaseguro que desea implementar en la optimización  
Valor Esperado

Seleccione los parámetros para el cálculo del costo de cada tipo de reaseguro, de acuerdo al principio de costeo seleccionado.

Excedente:  $\theta = 0,15$  Cuota\_Parte:  $\theta = 0,15$  XL:  $\theta = 0,3$  SL:  $\theta = 0,5$

Seleccione la medida de riesgo que desea implementar en la optimización  
VaR  $\alpha = 0,99$

Algoritmo a Implementar: NSGAI  
# de Evaluaciones: 5000

**Parámetros para Discretización**  
Defina los siguientes valores necesarios para la discretización de las funciones de distribución.

Valor Expuesto por Riesgo	Afectación al VE	Pérdidas Totales
Valor Mínimo: 0	Valor Mínimo: 0	Valor Mínimo: 0
Valor Máximo: 100	Valor Máximo: 1	Valor Máximo: 1000
Paso: 1		Paso: 1

**Ejecutar**

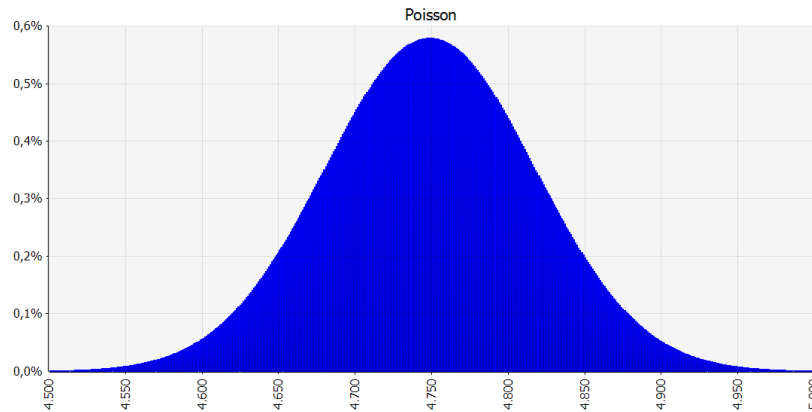


# Insurance portfolio

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- ▶ Fire risk of a large Colombian insurance company
- ▶ 256188 insurance contracts over 5 years
- ▶ Fitting of appropriate distributions.
  
- ▶ Example: Number of claims  $N$

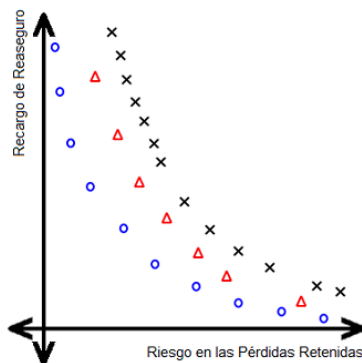
*Poisson* ( $\lambda = 4748.9$ )



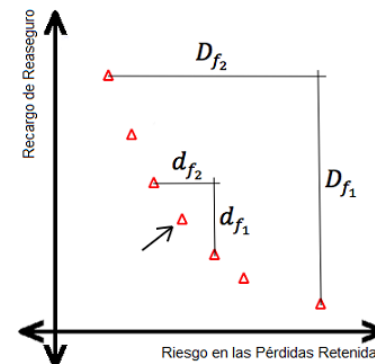
# Comparison against NSGA-II

- ▶ NSGA-II: well known multiobjective genetic algorithm
- ▶ Previously used for this problem by Oesterreicher et al (2006) and Mitschele et al (2007)
- ▶ Crossover with *Simulated Binary Crossover* (Agrawal et al 2000)
- ▶ Population size 50, number of evaluations 5000

NSGA II selection



NSGA II crowding distance



# Comparison against NSGA-II

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- ▶ 10 runs of each method
- ▶ Comparison of dominance by pairs using C metric by Jaszekiewicz (2004):

$$C(A, B) = \frac{|\{b \in B \mid \exists a \in A: a \prec b\}|}{|B|}$$

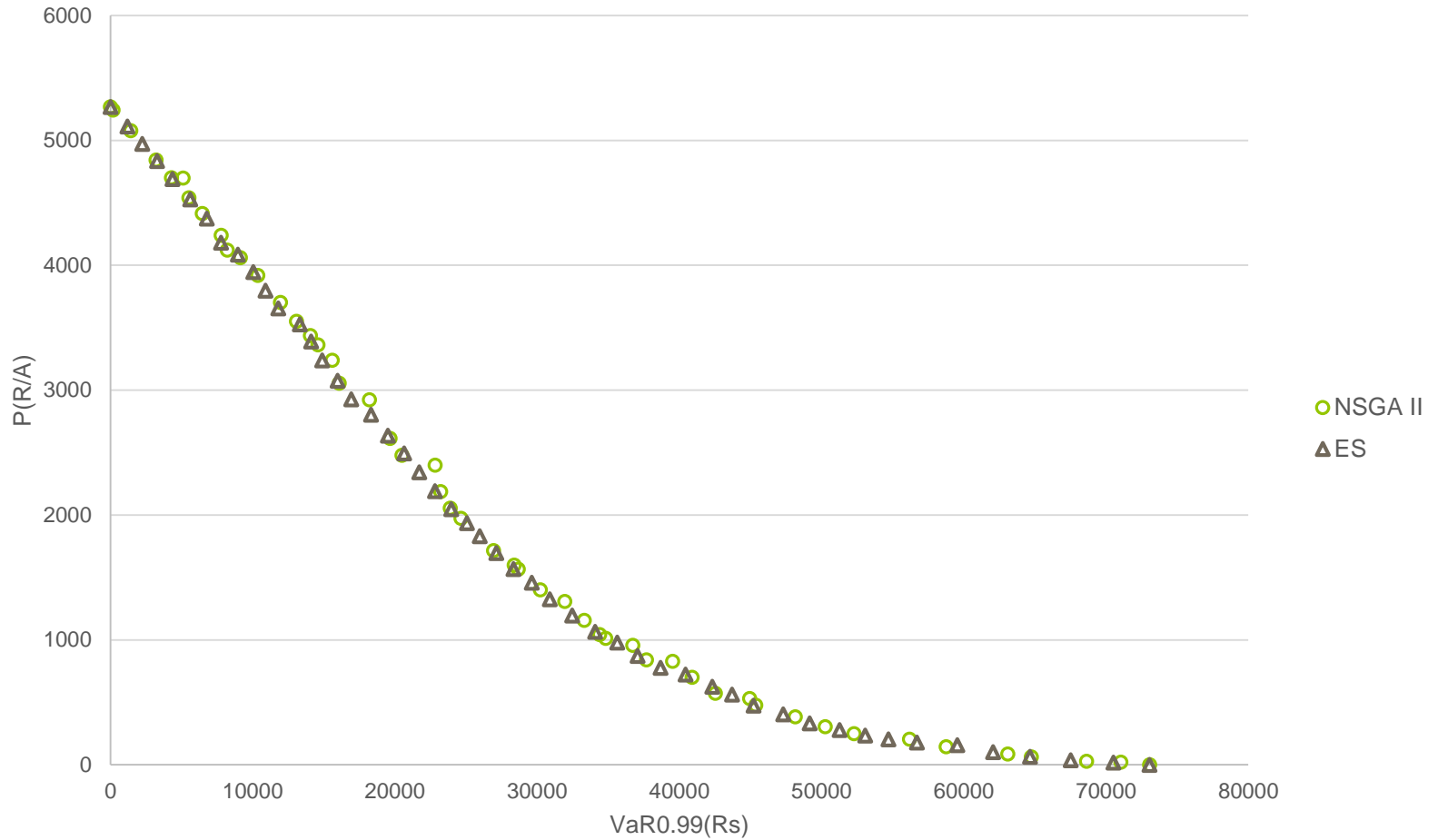
- ▶ Scott (1995) spacing metric:

$$s^2(PF) = \frac{1}{49} \sum_{i=1}^{50} (\bar{d} - d_i)^2$$

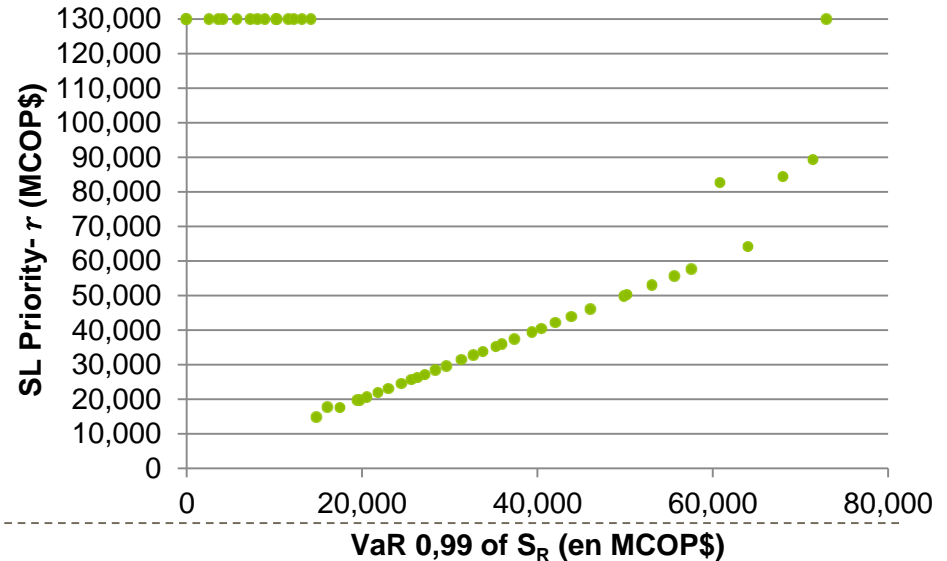
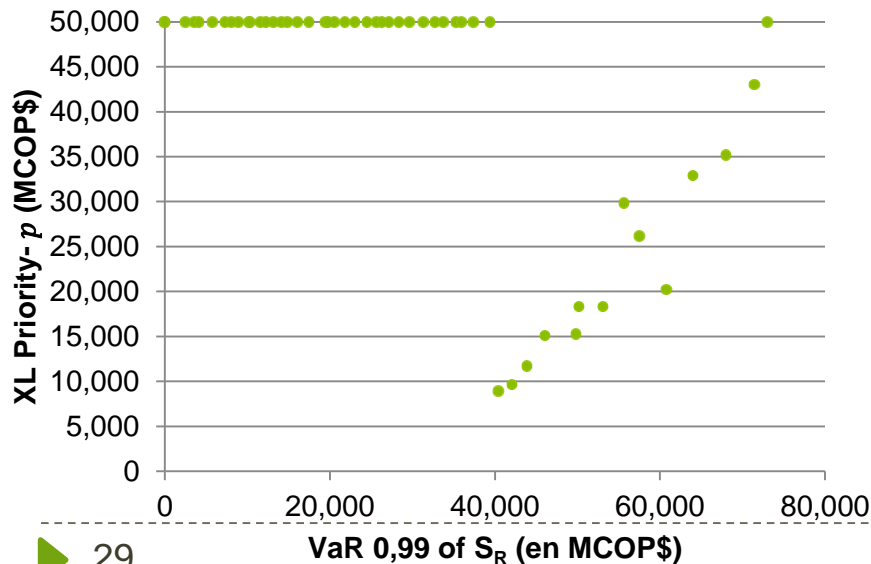
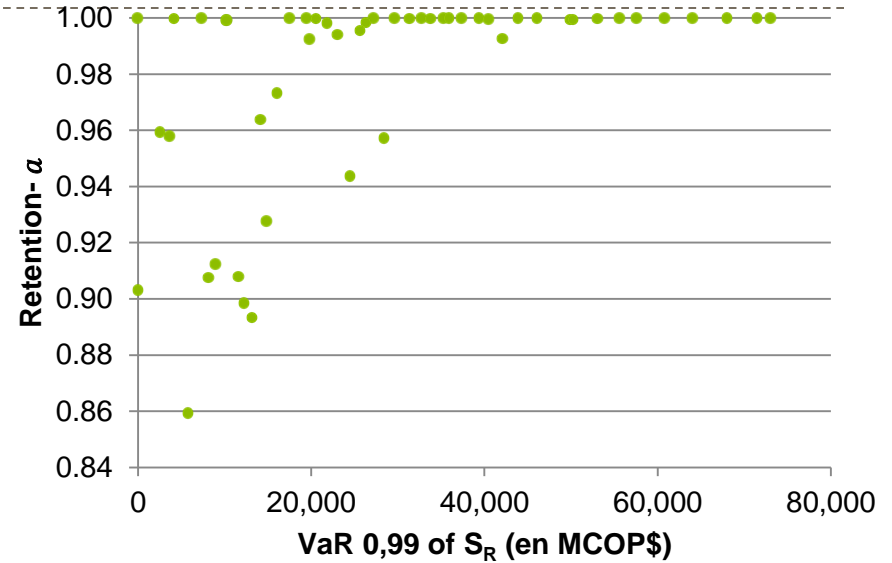
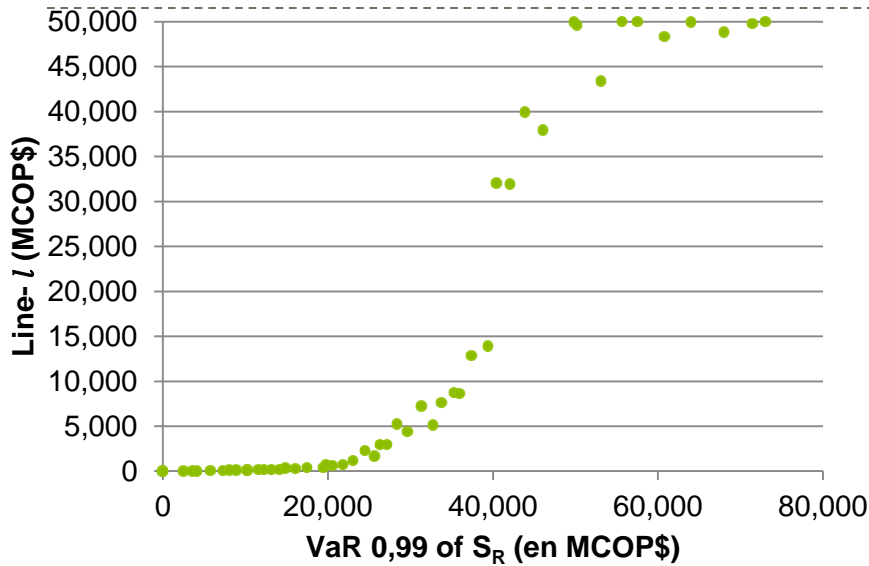
Metric	NSGA II	ES
Average time (s)	26552	23575
Average spacing	1.6E-04	2.4E-05
Average C(ES,NSGA-II)	15.58%	
Average C(NSGA-II,ES)	13.24%	

# Approximated efficient frontiers

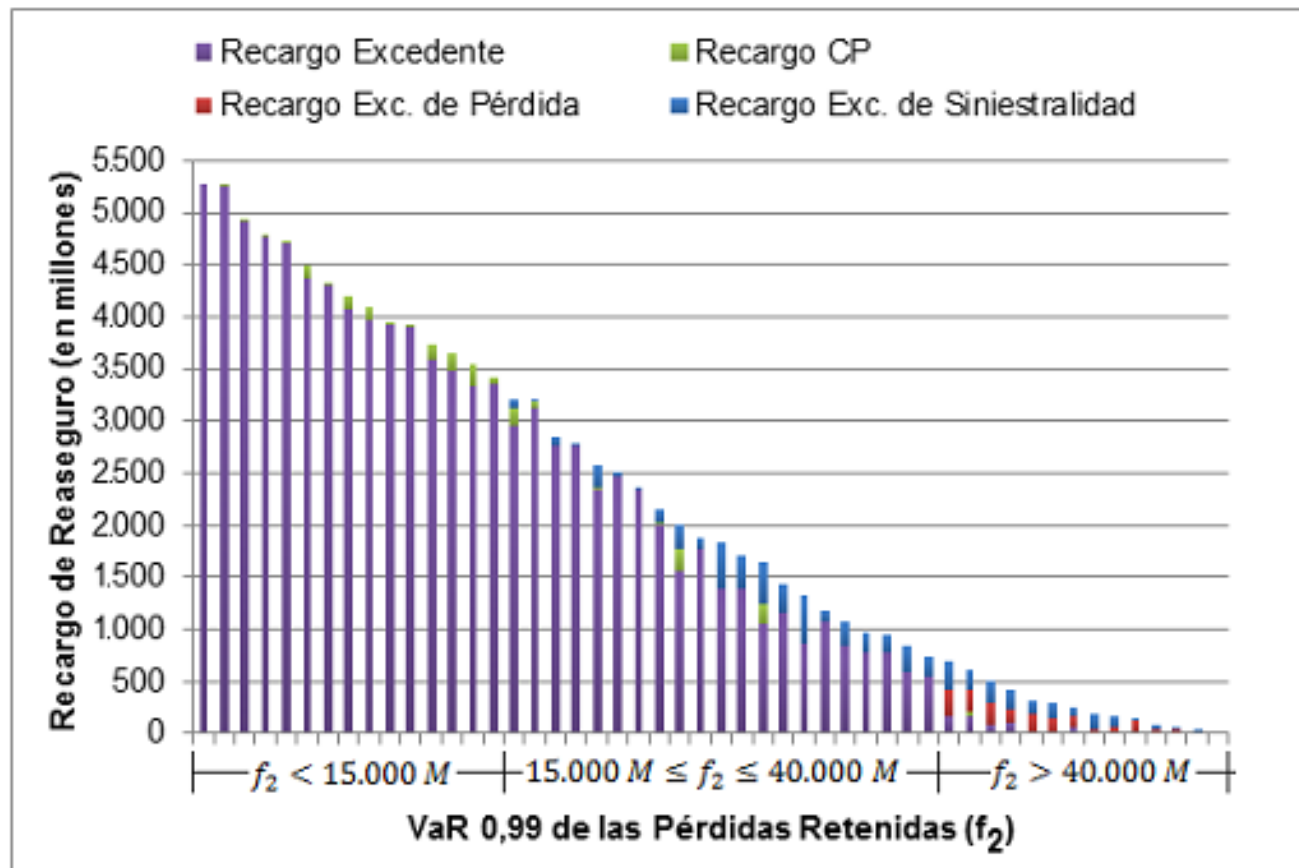
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# Structure of the solutions in the efficient frontier



# Premium distribution of the points in the efficient frontier



# Conclusions

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- ▶ A complex bi-objective reinsurance optimization problem
  - ▶ Minimize the cost of the reinsurance
  - ▶ Minimize the risk of the insurance portfolio
- ▶ Simple evolutionary strategy
- ▶ Unveils the trade-off of the objectives
- ▶ Outperforms NSGA-II
- ▶ Diverse set of reinsurance contracts, different from those traditionally used
- ▶ TO DO: explore opportunities to reduce the running time of the method



**Merci, gracias, thanks**  
**¿Questions? Comments**

Contact info: [juan.villegas@udea.edu.co](mailto:juan.villegas@udea.edu.co)