An evolutionary strategy for multiobjective reinsurance optimization - A case study

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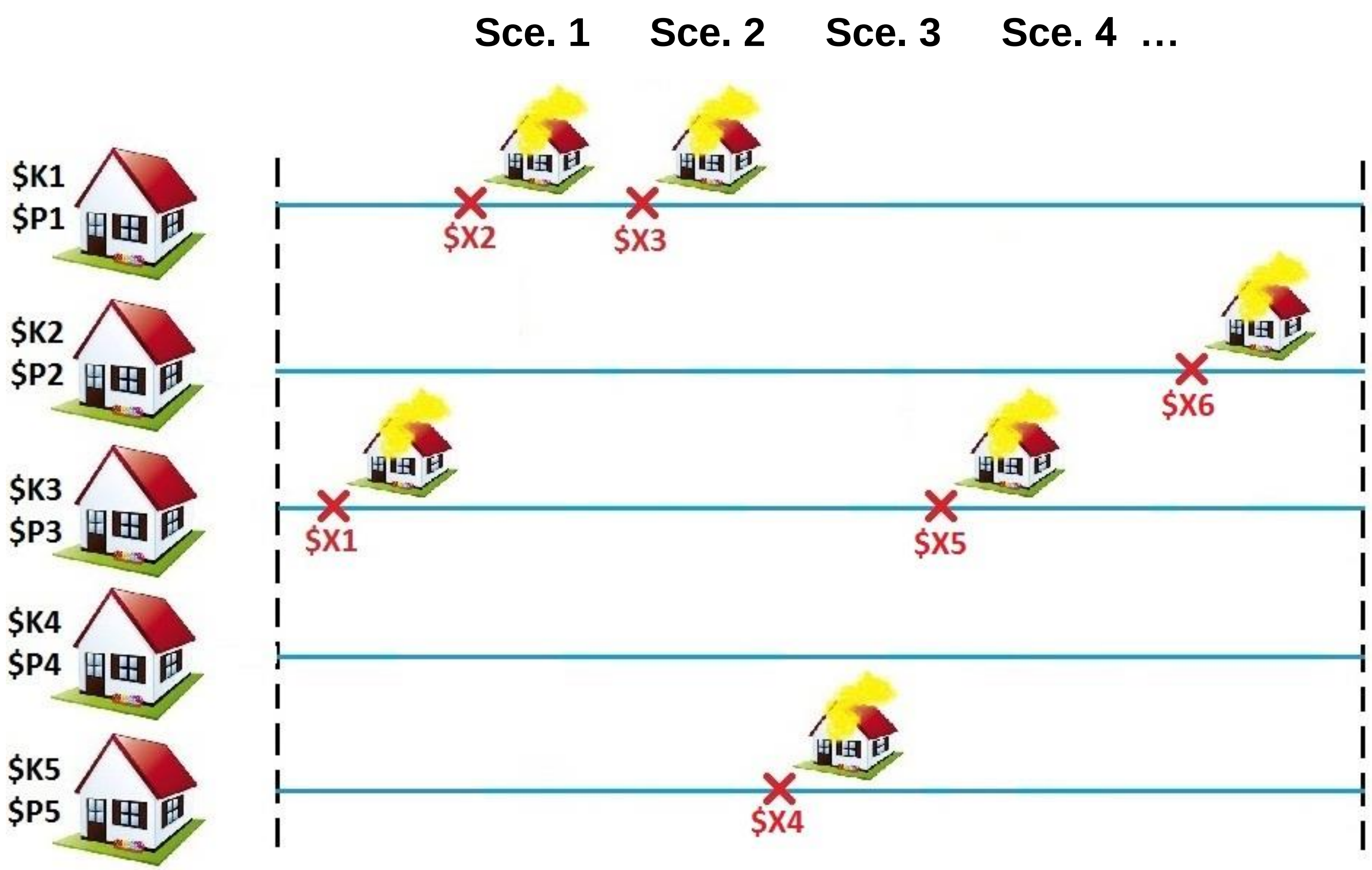
Agenda

1. Introduction and problem definition
2. Solution method
3. Case study

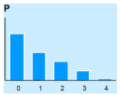
Introduction and problem definition

Insurance business



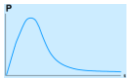
X_i: *value of claim i*

Risk of an insurance portfolio

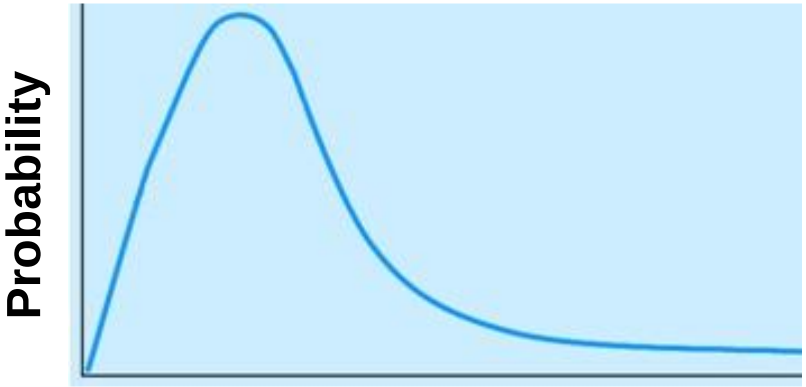


de claims

$$S = \sum_{i=1}^{N} X_i$$



N, X_i and S are random variables



S: Cumulated losses of the insurance company

Reinsurance contracts (R/A)

The insurance company transfers portions of its risk to other companies (the reinsurer)

$$S = S_R + S_C$$

$S_R: retained\ risk$

$S_C: ceded\ risk$

Risk transfer functions

$$S_C = F_1\left(\sum_{i=1}^{N} F_2(X_i)\right) \longrightarrow S_R = S - S_C$$

$F_1: global\ transfer\ function$

$F_2: local\ transfer\ function$

Reinsurance premium (cost of the reinsurance):

$$P_{R/A} = \Pi(S_C)$$

Π: Pricing principle

Reinsurance optimization problem (ROP)

Financial result of the insurance company

$$U = P - S + S_C - \Pi(S_C)$$

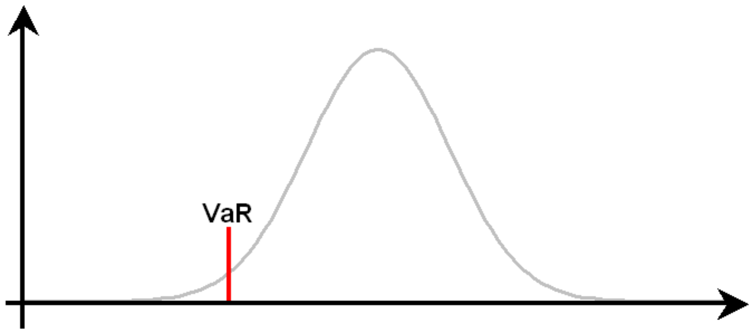
P: *insurance premiums,*

S: *total losses,*

S_C: *ceded losses*

$\Pi(S_C)$: *reinsurance premium*

ROP: Find F_1 *and* F_2 that <u>maximize</u> $E(U)$ and <u>minimize</u> a measure of risk of U, for example $VaR_{0.99}(U)$



Literature overview

Reference	Method	Comments
Oesterreicher et al (2006)	NSGA-II and $\epsilon - MOEA$	First MOEAs for R/A optimization
Mitschele et al (2007)	NSGA-II and $\epsilon - MOEA$	Practical application fire risks
Carmona et al (2013)	Population based incremental learning (PBIL)	Layered reinsurance - Scalarization approach
Carmona et al (2014)	PSO and differential evolution (DE)	Layered reinsurance - Scalarization approach
Salcedo Sanz et al (2014)	Evolutionary strategy and PSO	Mono-objective optimization, simple reinsurance contracts
Brown et al (2014)	Parallel multiobjective PBIL	Computing time reduction, no scalarization
Carmona et al (2015)	Parallel multiobjective DE	Computing time reduction, no scalarization

Literature overview

Reference	Method	Comments
Oesterreicher et al (2006)	NSGA-II and $\epsilon - MOEA$	First MOEAs for R/A optimization, high mutation probabilities
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We propose a multiobjective evolutionary strategy to solve the ROP

Classical reinsurance contracts/ Proportional reinsurance

- Based on the individual losses S:

$$S = \sum_{i=1}^{N} X_i$$

- **Proportional reinsurance**: A portion of the loss is transferred to the reinsurer

Quota share reinsurance (CP)

$$X_{i_C} = (1 - a_i)X_i \longrightarrow S_C = \sum_{i=1}^{N} X_{i_C}$$

a_i: <u>Retention value (usually the same for all *i*)</u>

Example

$$X_i = 100$$
$$a_i = 0.25$$

$$X_{ic} = (1 - 0.25)100 = 75$$

Classical reinsurance contracts/ Proportional reinsurance

- Based on the individual losses S:

$$S = \sum_{i=1}^{N} X_i$$

- **Proportional reinsurance:** A portion of the loss is transferred to the reinsurer

Surplus reinsurance (Exc)

$$X_{i_C} = (1 - a_i)X_i \quad \text{with} \quad a_i = \min\left(\frac{L}{K_i}, 1\right)$$

***L*: Line of the reinsurance**

K_i: Insured value of loss i

Example

$$X_i = 100$$
$$L = 90$$
$$K_i = 180$$
$$a_i = \min\left(\frac{90}{180}, 1\right) = 0.5$$
$$X_{ic} = (1 - 0.5)100 = 50$$

Classical reinsurance contracts/ Proportional reinsurance

- Based on the individual losses S:

$$S = \sum_{i=1}^{N} X_i$$

- **Non proportional reinsurance**: Truncates the losses

Excess of Loss reinsurance (XL)

$$X_{i_C} = max(X_i - P, 0) \longrightarrow S_C = \sum_{i=1}^{N} X_{i_C}$$

***P*: Priority of the excess of loss**

Example

$$X_i = 100$$
$$P = 90$$
$$X_{ic} = \max(100 - 90{,}0) = 10$$

Classical reinsurance contracts/ Proportional reinsurance

- Based on the individual losses S:

$$S = \sum_{i=1}^{N} X_i$$

- **Non proportional reinsurance**: Truncates the losses

Stop Loss reinsurance (SL): applies to the aggregate losses

$$S_C = \max(S - R, 0)$$

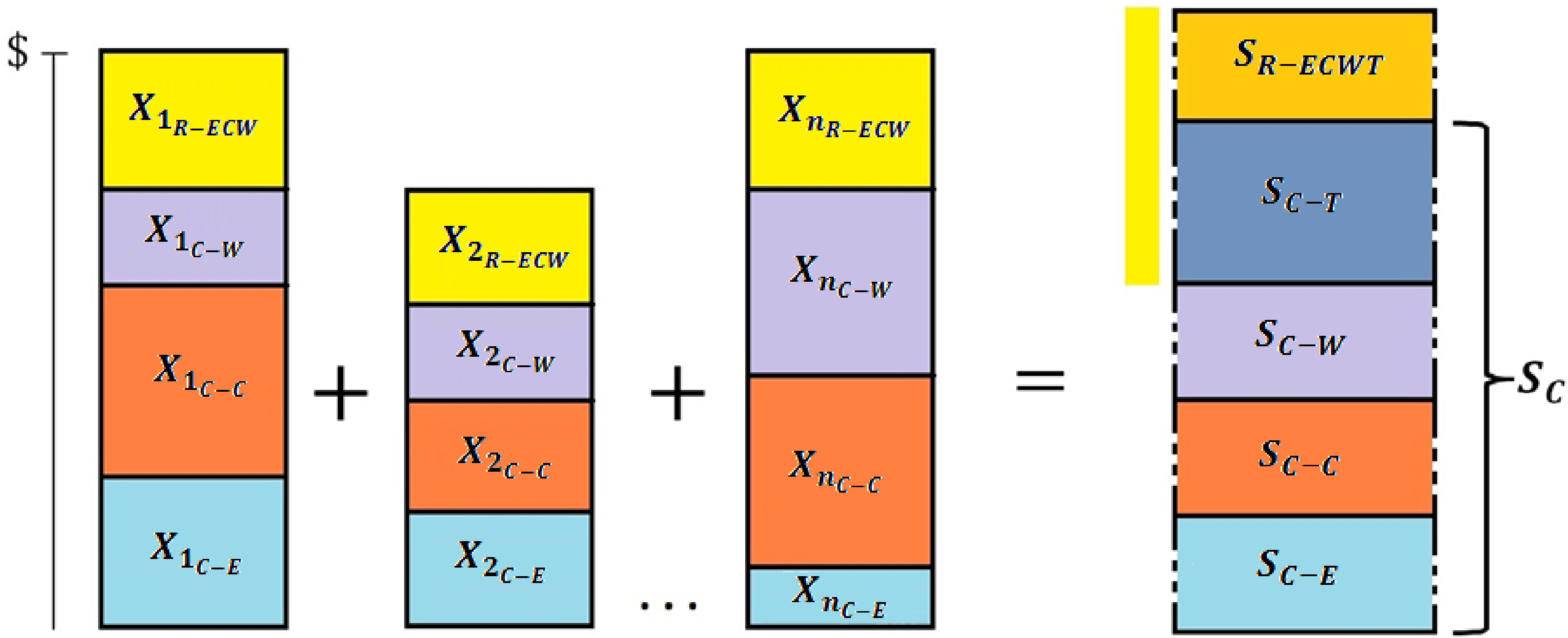
***R*: Priority of the stop loss**

Example

$$S = 1000$$
$$R = 800$$
$$S_c = \max(1000 - 800, 0) = 200$$

Ceded and retained risk

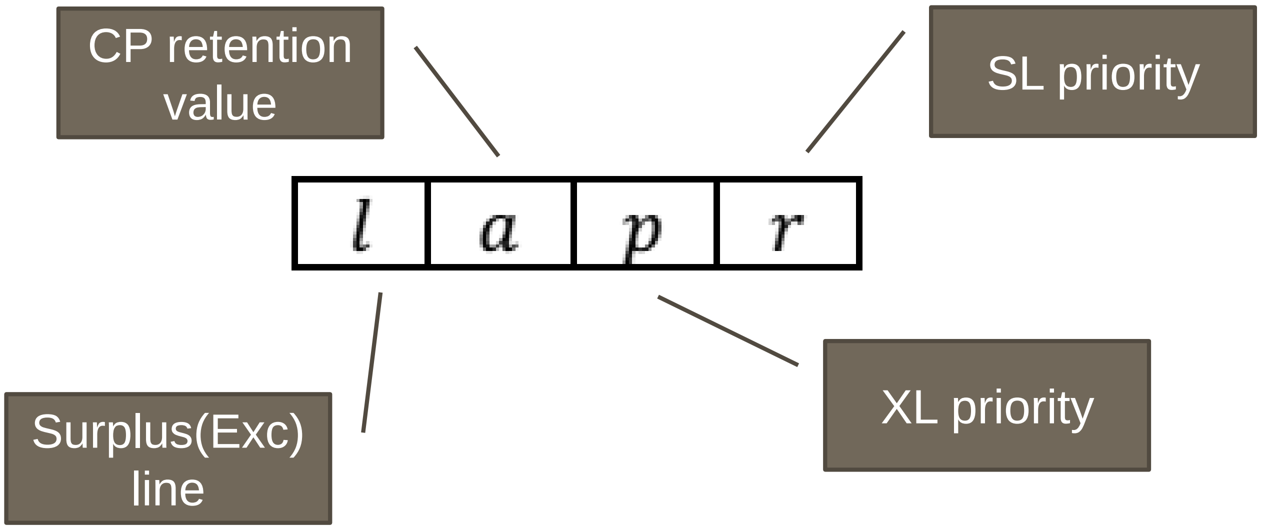
$$S_C = S_{C-Exc} + S_{C-CP} + S_{C-XL} + S_{C-SL}$$



Solution method: Evolutionary strategy

Solution representation

- Solution representation



Objective function

$$U = P - S + S_c - \Pi(S_c)$$

- Minimize $\Pi(S_c)$

$$f_1(l, a, p, r) = P_{R/A} = \Pi(S_c) = (1 + \lambda)E(S_c)$$

- Minimize the risk of the retained losses ($S_R = S - S_C$)

$$f_2(l, a, p, r) = VaR_\alpha(S_R) = F^{-1}(\alpha)$$

Objective function calculation: since N, X_i and S are random variables

- Probabilistic modeling of the insurance portfolio

Individual loss for a claim , **X:** $F_X(x) = \int_{k=0}^{\infty} F_Z\left(\frac{x}{k}\right) c(k)\, \partial k$

Retained loss after reinsurance Exc, $\mathbf{X_{R\text{-}E}}$ **:** $F_{X_{R-E}}(x) = \int_{k=0}^{L} F_Z\left(\frac{x}{k}\right) c(k)\, \partial k + \bar{C}(\boldsymbol{L}) F_Z\left(\frac{x}{\boldsymbol{L}}\right)$

Retained loss after reinsurance Exc and CP, $\mathbf{X_{R\text{-}EC}}$ **:** $F_{X_{R-EC}}(x) = \int_{k=0}^{L} F_Z\left(\frac{x}{\boldsymbol{a}k}\right) c(k)\, \partial k + \bar{C}(\boldsymbol{L}) F_Z\left(\frac{x}{\boldsymbol{aL}}\right)$

Retained loss after E, CP and XL, $\mathbf{X_{R\text{-}ECX}}$ **:**

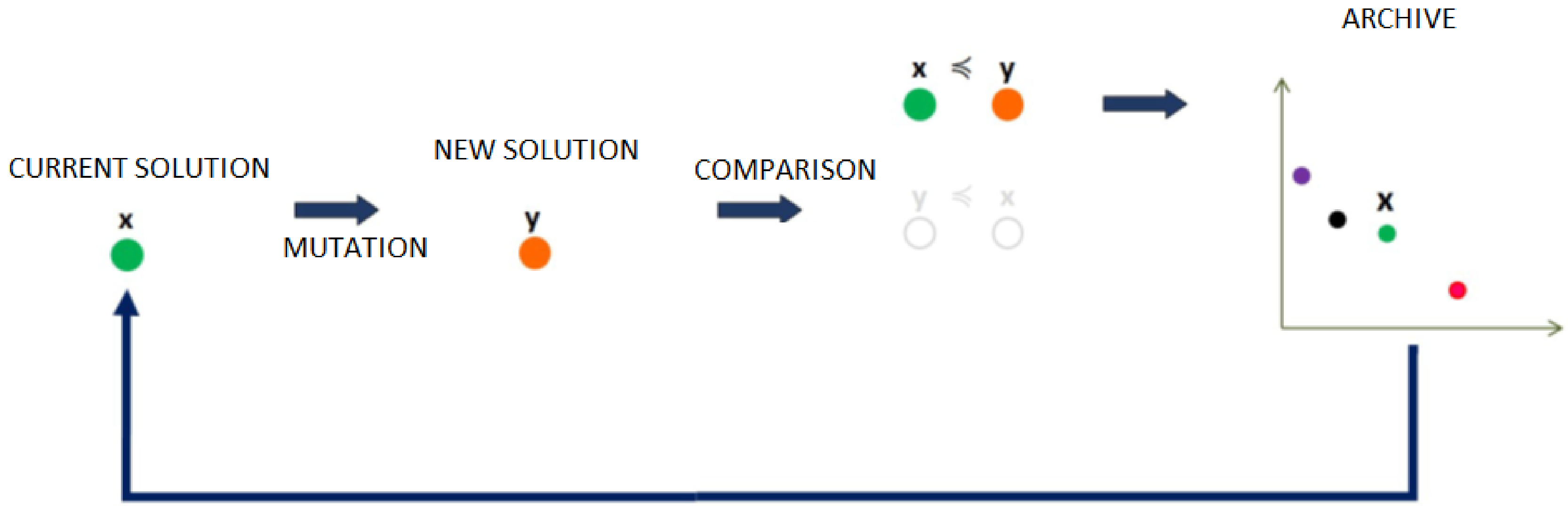
$$F_{X_{R-ECX}}(x) = \begin{cases} \int_{k=0}^{L} F_Z\left(\frac{x}{\boldsymbol{a}k}\right) c(k)\, \partial k + \bar{C}(\boldsymbol{L}) F_Z\left(\frac{x}{\boldsymbol{aL}}\right); & si\ x < \boldsymbol{P} \\ 1; & si\ x \geq \boldsymbol{P} \end{cases}$$

Aggregated losses after Exc, CP y XL, $\mathbf{S_{R\text{-}ECX}}$:

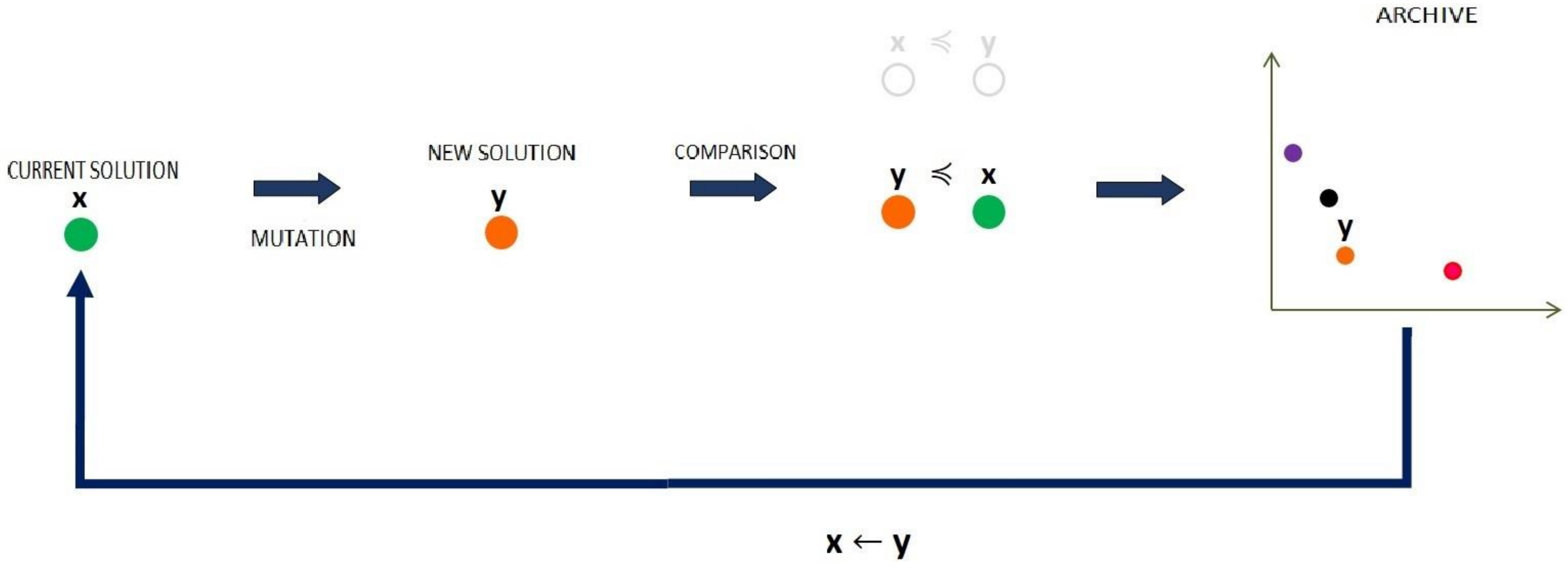
Aggregated retained losses after Exc, CP, XL y SL, $\mathbf{S_{R\text{-}ECXS}}$ $F_{S_{ECXS}}(x) = \begin{cases} F_{S_{ECX}}(x); & si\ x < \boldsymbol{R} \\ 1; & si\ x \geq \boldsymbol{R} \end{cases}$

We use numerical integration after fitting appropriate statistical distributions

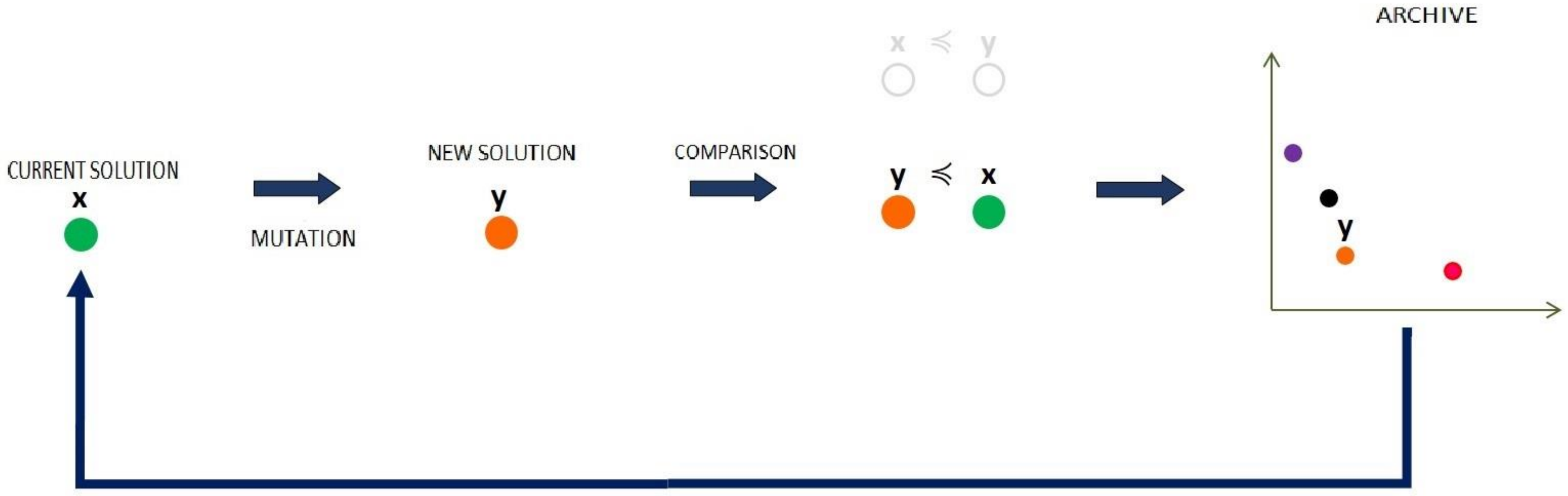
A simple Pareto archived evolutionary strategy



A simple Pareto archived evolutionary strategy (ES)



A simple Pareto archived evolutionary strategy (ES)



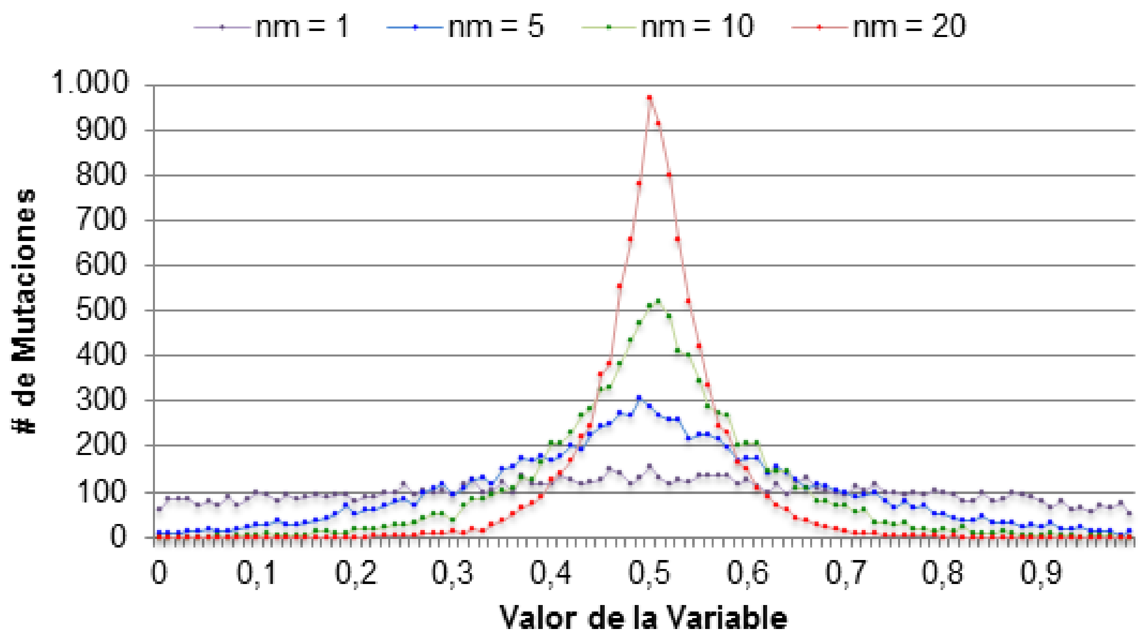
RESTART FROM A NEW SOLUTION IN THE ARCHIVE AFTER 20 GENERATONS WITHOUT IMPROVEMENT OF THE FRONTIER

Mutation operator

- Parameter based mutation operator (Deb & Goyal, 1996)

$$\bar{\delta} = \begin{cases} [2u + (1-2u)(1-\delta)^{n_m+1}]^{\frac{1}{n_m+1}} - 1, & \text{si } u \le 0.5 \\ 1 - [2(1-u) + 2(u-0.5)(1-\delta)^{n_m+1}]^{\frac{1}{n_m+1}}, & \text{si } u > 0.5 \end{cases}$$

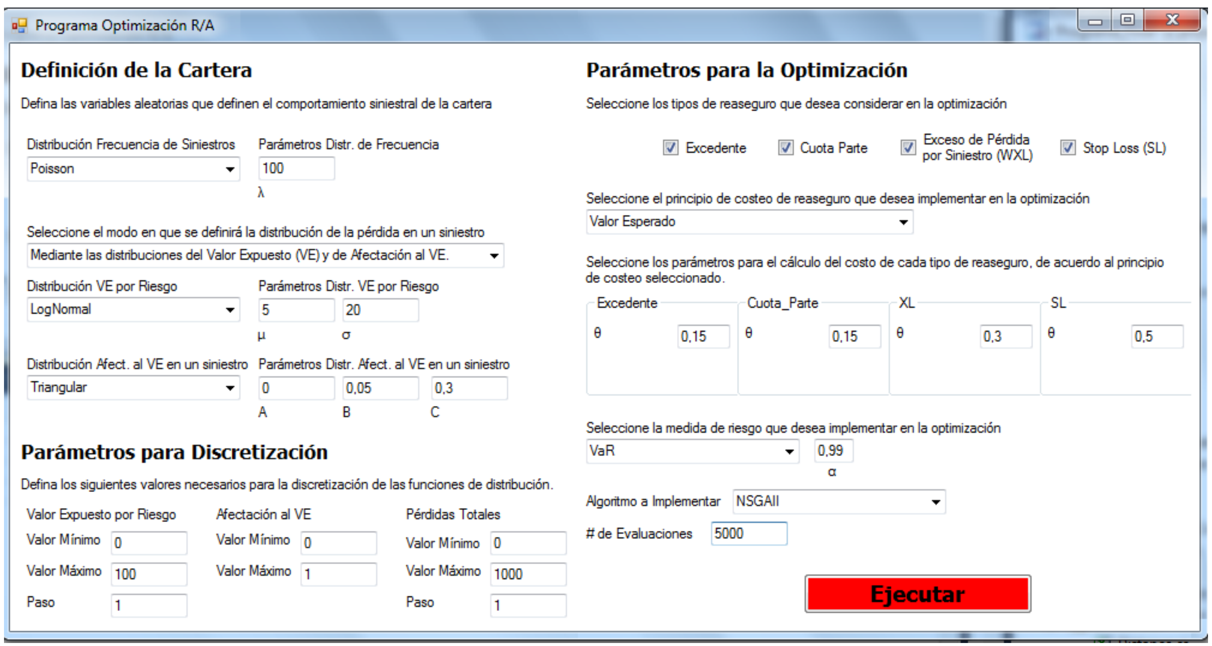
$$\delta = \min[(x - x_m), (x_M - x)] \qquad y' = y + \bar{\delta}\Delta_{max} \qquad \Delta_{max} = x_M - x_m$$



Case study

Implementation and case study

- Evolutionary strategy (ES) implemented in C#
- Decision support system for the Company
  - Results for mutation index $n_m = 10$, archive size: 50 and number of iterations of ES: 5000

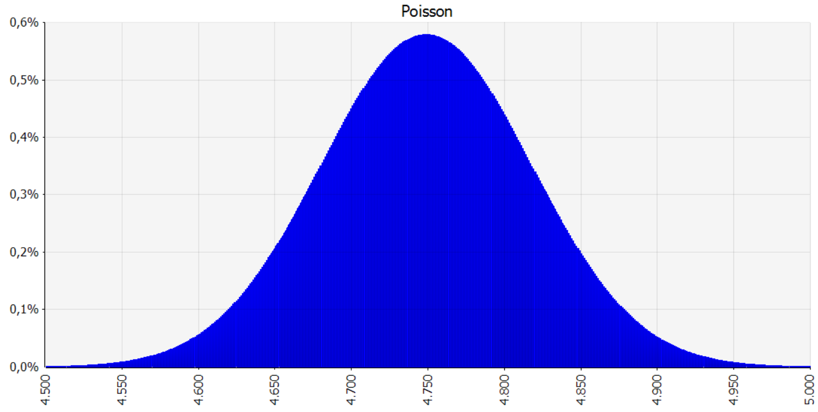


Insurance portfolio

- Fire risk of a large Colombian insurance company
- 256188 insurance contracts over 5 years
- Fitting of appropriate distributions.

- Example: Number of claims N

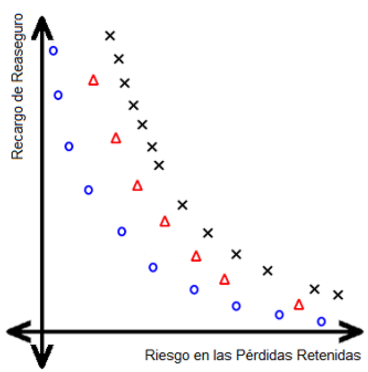
$$Poisson\ (\lambda = 4748.9)$$



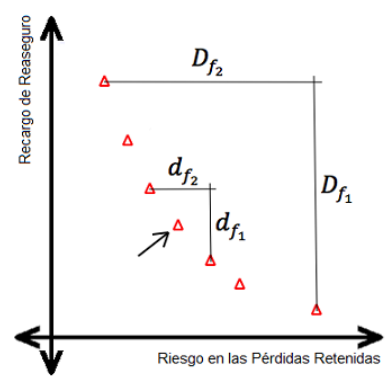
Comparison against NSGA-II

- NSGA-II: well known multiobjective genetic algorithm
- Previously used for this problem by Oesterreicher et al (2006) and Mitschele et al (2007)
- Crossover with *Simulated Binary Crossover* (Agrawal et al 2000)
- Population size 50, number of evaluations 5000

NSGA II selection



NSGA II crowding distance



Comparison against NSGA-II

- 10 runs of each method
- Comparison of dominance by pairs using C metric by Jaszkiewicz (2004):

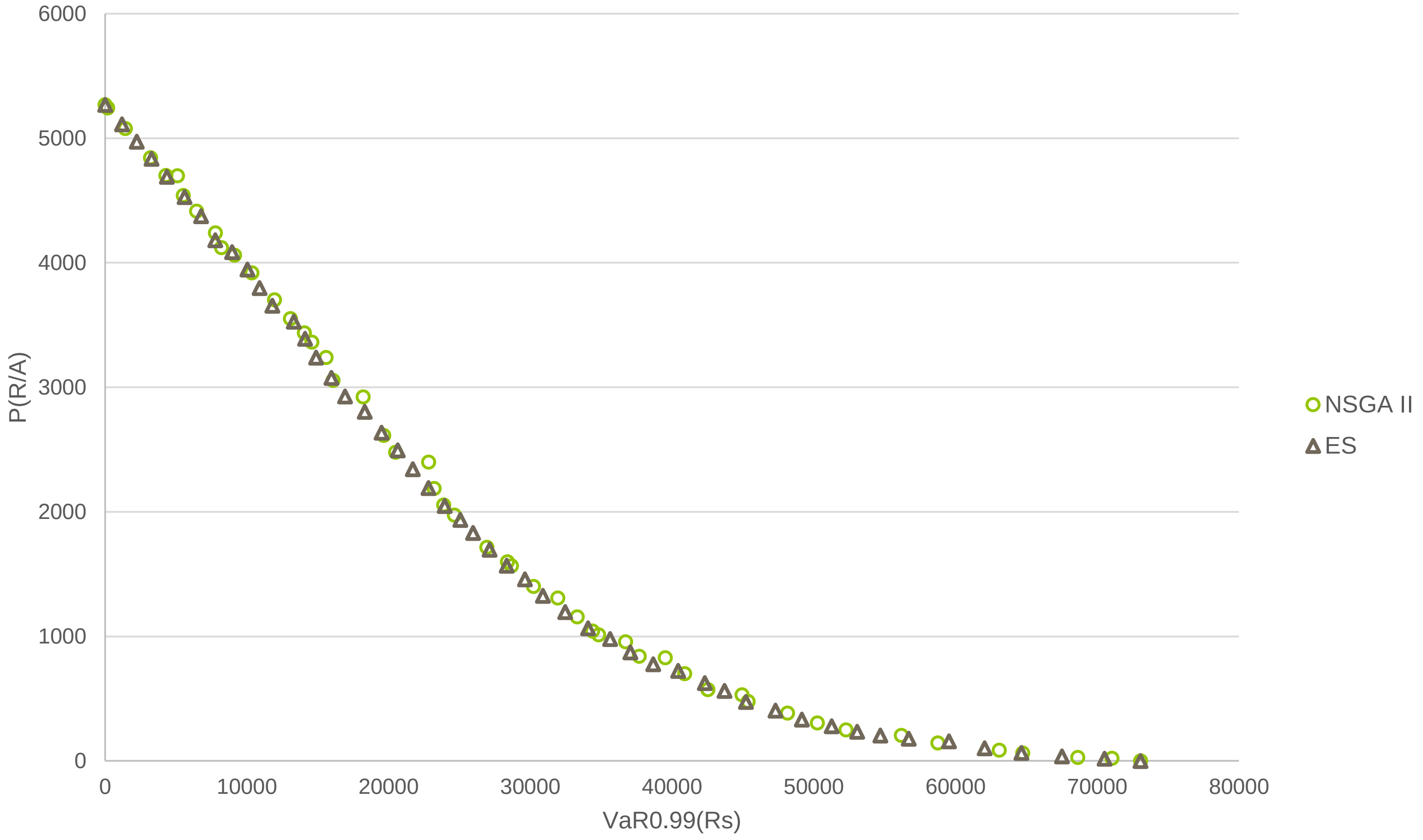
$$C(A,B) = \frac{|\{b \in B \mid \exists a \in A : a \preceq b\}|}{|B|}$$

- Scott (1995) spacing metric:

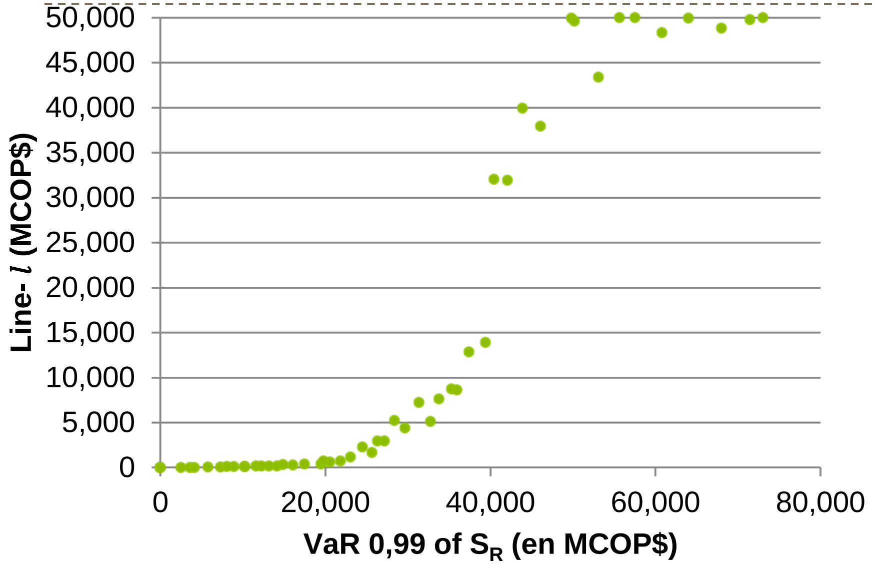
$$s^2(PF) = \frac{1}{49}\sum_{i=1}^{50}(\bar{d} - d_i)^2$$

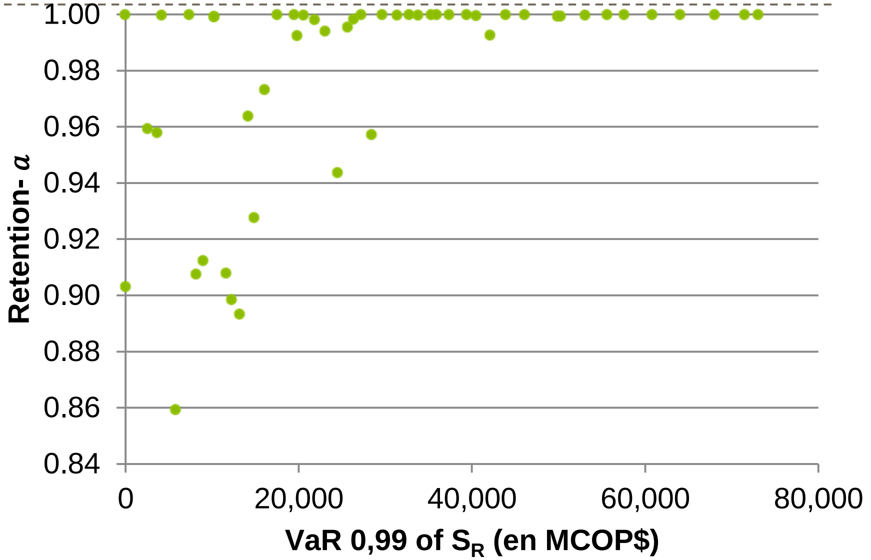
Metric	NSGA II	ES
Average time (s)	26552	23575
Average spacing	1.6E-04	2.4E-05
Average C(ES,NSGA-II)	15.58%	
Average C(NSGA-II,ES)	13.24%	

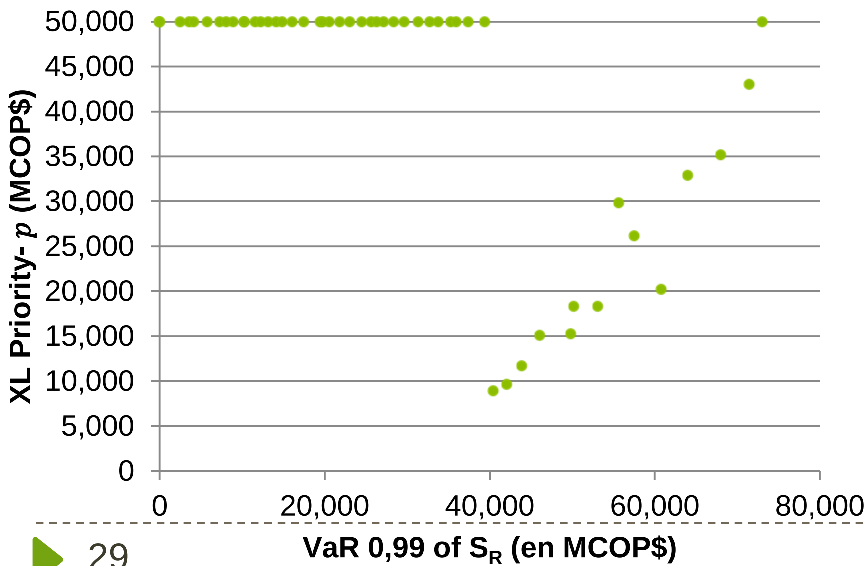
Approximated efficient frontiers

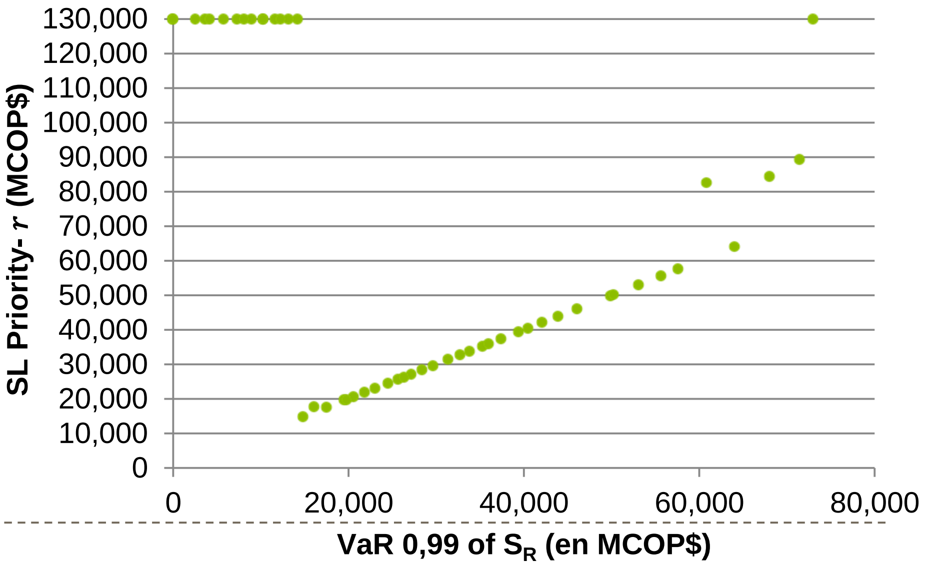


Structure of the solutions in the efficient frontier

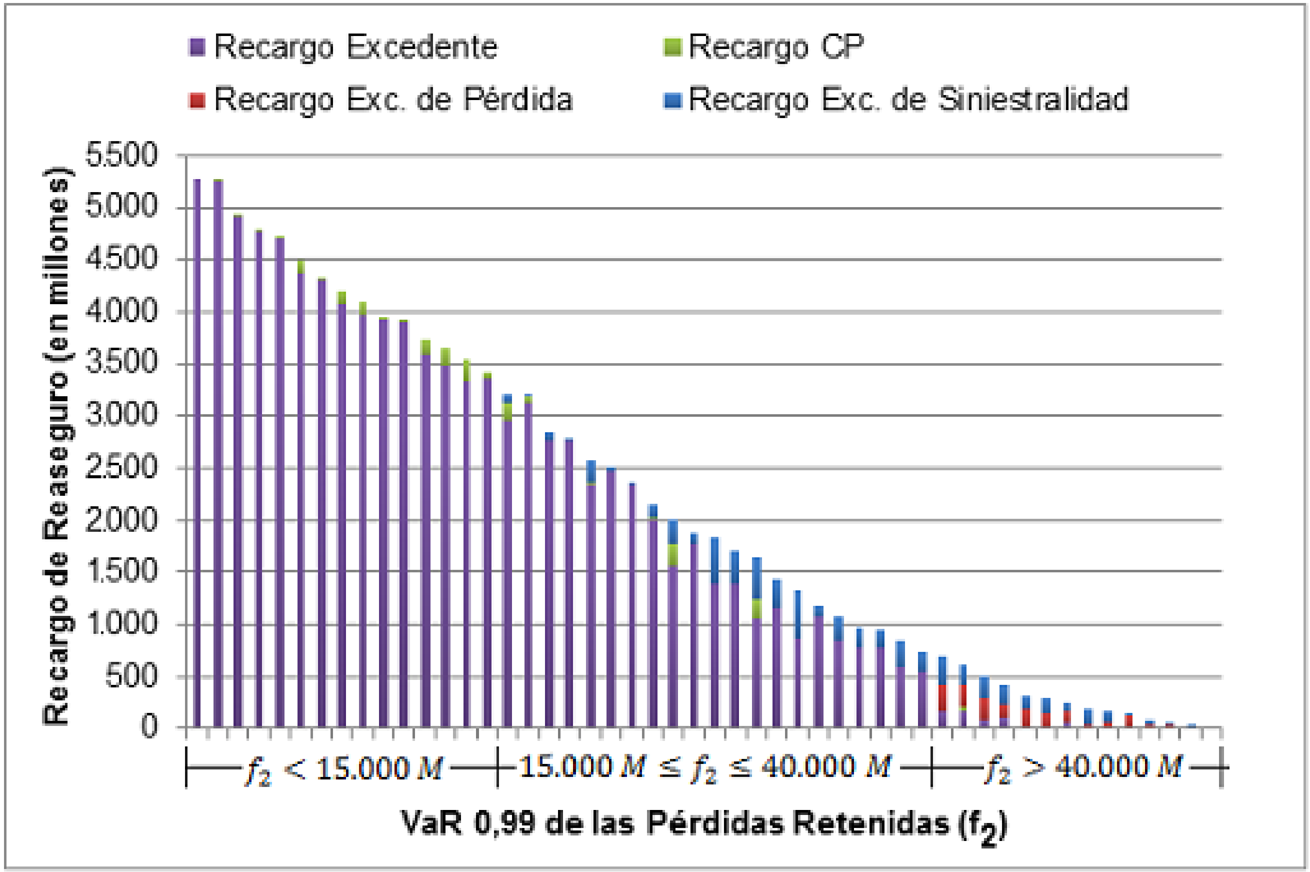








Premium distribution of the points in the efficient frontier



Conclusions

- A complex bi-objective reinsurance optimization problem
  - Minimize the cost of the reinsurance
  - Minimize the risk of the insurance portfolio
- Simple evolutionary strategy
- Unveils the trade-off of the objectives
- Outperforms NSGA-II
- Diverse set of reinsurance contracts, different from those traditionally used
- TO DO: explore opportunities to reduce the running time of the method

Merci, gracias, thanks ¿Questions? Comments

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