LARIS
Laboratoire Angevin de Recherche
en Ingénierie des Systèmes
EA7315

Ph.D. Proposal

Thesis subject

Set-based approaches for multi-agent simultaneous localization and mapping.

Context

The Simultaneous Localization and Mapping (SLAM) is one of the essential tasks in autonomous mobile robotics. In this task, the robot localizes itself and creates a map of its environment by using proprioceptive sensors (encoders, IMUs ...) and exteroceptive sensors (GNSS, Sonar ...) identifying landmarks and estimating its distance to these points. Figure 1 illustrates the steps of this task.

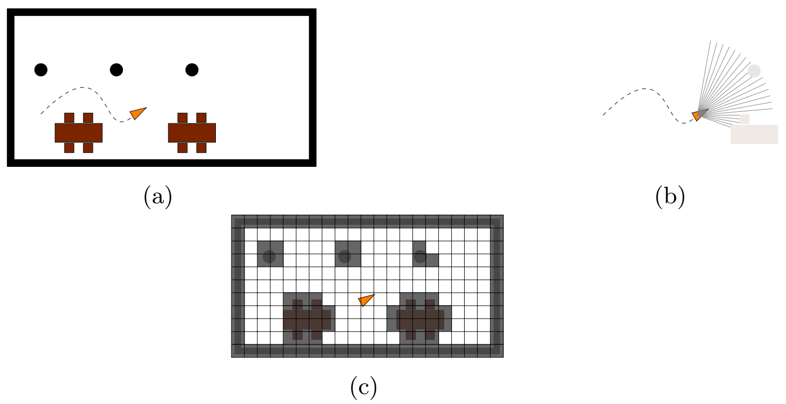

FIGURE 1 – The robot is deployed in a room (a) where it moves and measures some information about its path (dashed line) and its distance to the obstacles using telemeters (b). By registering the found obstacles, the robot creates an occupation grid (c) and it estimates its position with respect to this map.

The majority of the estimation methods in the literature supposes a gaussien character of the measures done by the sensors cited. By exploiting this kind of property, statistical filters such as EKF (Extended Kalman Filter) or particle filters were developed as well as SLAM (Simultaneous Localization and Mapping) methods that mix and match them [Thr+04]. This approach received great attention in the last decades and these algorithms are well established and used in a plethora of application, including some in the everyday life, such as domestic robot vacuum cleaners, mobile telephones for photo-taking and also in human-machine interfaces as in virtual reality goggles. Although largely used, those methods can have some caveats if the system presents non-linear dynamics or non-gaussian distributions. An approximation of the non-gaussian distribution by a gaussian one, as in Figure 2, can provoke a lack of convergence of EKF-like filters or even converge to an abnormal value.

Furthermore, even if the distributions could be approximated by gaussian ones, the classical statistical approaches estimate only the mean and variance of the distribution of the studied quantities. Therefore, we do not have any guarantee of the measurements and we can only use confidence intervals, such the one in Figure 3.

For the cited applications, the lack of guarantees do not necessarily lead to grave consequences, but for other applications, such as inspection, or driving autonomous vehicles in multi-modal environments, this gap can cause vital consequences. For this kind of operations, which need more reliability, it is necessary to master the uncertainties of the robot and its environment.

The set-based tools, such as the interval analysis [Jau+01], present a solution where the uncertainties are considered bounded and we can ignore their statistical distribution. For these tools, there are two main approaches.

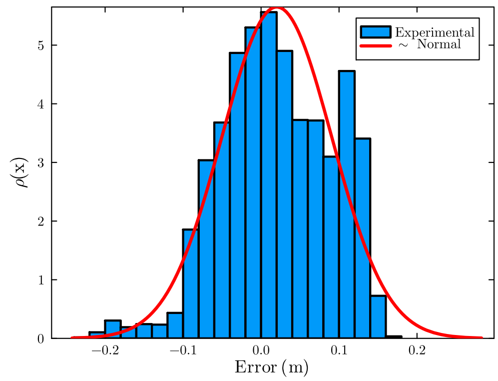

FIGURE 2 – Histogram of the measurement error of a telemeter and its normal distribution approximation.

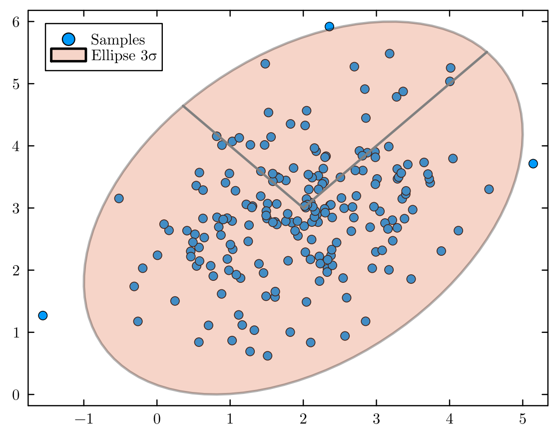

FIGURE 3 – Example of samples following a 2D normal law (representing the position of an object) and the associated 3σ ellipse.

In the first, the uncertainties are bounded, i.e., constrained to an uncertainty box and the estimation problems, particularly localization and mapping, can be described as CSP (Constraint Satisfaction Problem). We will name this approach as ≪CSP approach≫. By customizing operators called ≪contractors≫, the algorithms can minimize the volume of the box that surrounds the measurand. We can cite some examples of those interval-based SLAM algorithms such as in [Le +10], [Guy13] and [Mus+18]. In [Sli11], the author develops a robust version where it is impossible to respect all the constraints simultaneously and thus some constraints of the CSP must be relaxed.

Another approach is to use the basis of ≪Kalman≫-like algorithms, but using set formalism, as interval analysis, and calculating the optimal estimation bounds, e.g. [Lu+19] for IKFs (Interval Kalman Filters). We name this approach ≪set-based Kalman approach≫. In [LQ18], the authors assocate boxed particle filters and EIKF (Extended Interval Kalman Filter) for a guaranteed SLAM technique.

In theses two approaches, we can make two remarks. The first remark is in the use of boxes, which, because of their symmetry and the fact that the boxes are axis-aligned, facilitates the calculation of some operations over sets (sum, union, intersection etc), but on the other hand, increases the conservatism if we enclose an asymmetric shape or a symmetric one with edges that are not parallel to the axis of the box, some examples are represented in Figure 4.

The second remark focus in the ≪CSP approach≫. We see in the literature that the majority of the works are based on binary search recursive algorithms such as SIVIA (Set Inversion Via Interval Analysis) [Her+12], to create a tiling of boxes as in Figure 5. These algorithms are in general computationally intense and need for certain cases that the calculation be executed offline, what prevents their use in ≪real-time≫ applications.

(a) (b)

FIGURE 4 – Examples of sets where the use of intervals (boxes) increase the conservatism. (a) Square rotated by 30°. (b) A triangle.

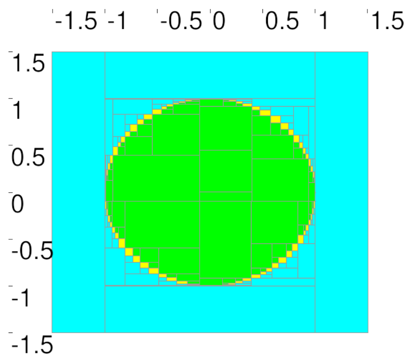

FIGURE 5 – Example of a tiling generated by SIVIA using inclusion tests (blue=not included, green=included, yellow=uncertain).

Objective

The objective of this thesis is to consider both the conservatism increase and calculation costs problems in two manners : first by using other set representations ; and the second by exploring distributed calculation methods.

In the first front, some works using other representations were initiated in the literature, for instance a method in the ≪set-based Kalman approach≫ where the zonotopes used in the estimation had their complexity reduced using contractors based on hyperplanes [BFJ25]. The shortcomings of zonotopes, and many other representations such as ellipses, is their lack of closeness in the intersection operation and also their symmetry.

In [Sco+16], a new representation called constrained zonotopes was proposed. The constrained zonotope enables the representation of non-symmetrical shapes (cf. Figure 6) and moreover, it is closed in the intersection operation. This approach is being used in a ever-increasing fashoin for non-linear state estimation and diagnostics [Reg+20], [Reg+24], [VKR25] and [Reg+25], however, as far we are concerned, it has not been used for SLAM, neither for ≪back-propagation≫ to refine the estimated sets.

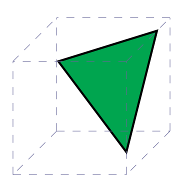

FIGURE 6 – Example of a constrained zonotope (in green), formed from the intersection of a zonotope (prism in gray) and a hyperplane.

For the distributed approaches, even though we find multi-agent SLAM in some works, multi-agent set-based SLAM seems still neglected. We propose to use approaches of the ≪Network Controlled Systems≫ community, such as ≪averaging≫, ≪gossiping≫ and ≪consensus≫, where agents exchange information to solve a common problem, as in [Bof+19], where the agents find cooperatively the solution of an optimization problem with a multi-agent version of Newton-Raphsons' algorithm. We find some examples of bounded consensus in the following works [BTV09 ; Fon+20]. In our case, the robots must interact, exchanging the available different parts of maps together with their respective uncertainties, and in the end, once the consensus achieved, we find a common map. In Figure 7, we see an illustration of a possible result, a collaborative map based on polygons (without an occupation grid).

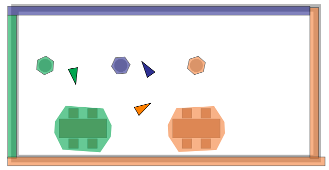

FIGURE 7 – Emulation of a polygon-based map, created by the contribution of different robots.

Other interesting aspects to study in the localization and mapping problem using other representations is the visibility among points in the map, initially studied in [Guy13] but with the interval analysis formalism, and the uncertainty propagation of the inter-agent distance in their respective maps, initially studied in [Dia25] also with the interval analysis formalism.

In what concerns the robots' movement during SLAM, when we free ourselves from occupation grids for representing the obstacles and parts of the map, other means of planning must be developed. A lead is to use the tools from convex optimization [BV04], so we can integrate vehicle dynamics and spatial constraints (obstacles), creating this way an active SLAM algorithm based on a set-based predictive control [LHD06 ; PS23].

Methodology and expected outcomes

This thesis has as main objective the study of set-based approaches for the multi-agent SLAM. In order to well appreciate the subject and confront its complexity, we propose a complexity increasing methodology :

In a first time, the person will work in the mapping and localization problem with a single robot. This way, the person will concentrate in formalizing these problems, exploring them initially detached and then together. A first approach is to treat the mapping isolated, where we suppose the position and orientation of the robot is known, directly or by using some preexisting localization algorithm. Alternatively, the focus could be the localization problem, supposing a known map. And then, we study the integration of the two parts for a complete SLAM.

Once the problem with a single robot is well defined, we will explore the multi-agent aspect by extrapolating the results of the previous step. Again, we can attack each problem separately and then together.

The person will explore the cited aspects depending on their profile and aptitudes. The studied and developed algorithm will be validated in simulation and ideally under the laboratory robotics platform.

Profile

To apply you should have a Master 2 or equivalent diploma.

- Competence in english required
- Knowledge in mobile robotics and localization algorithms required
- Knowledge in interval analysis and set-based calculation preferable
- Development skills in a high-level language Python/MATLAB/Julia required
- Development skills in a low-level language C/C++ required
- Development tools : git, ROS2 preferable

The person should demonstrate autonomy and initiative.

Laboratory

The thesis will take place at the Laboratoire Angevin de Recherche en Ingénierie des Systèmes (LARIS). The LARIS laboratory is a multidisciplinary research unit in Science and Technology belonging to the University of Angers (UA). The laboratory brings together researchers of several faculties and schools of the territory (`https://laris.univ-angers.fr/en/homepage.html`).

More specifically, research will be done in collaboration with researchers of the research axis ≪robotics and set-based methods≫ of the DSO (Dynamic Systems and Optimization) group (`https://laris.univ-angers.fr/en/laboratory/dynamic-systems-and-optimization-group.html`).

Funding

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Application

To apply for the position visit the official announcement :
`https://amethis.doctorat.org/amethis-client/prd/consulter/offre/2938`

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